reinforced concrete construction

Concrete Construction
- cast-in-place
- tilt-up
- prestressing
- post-tensioning

Concrete Materials
- low strength to weight ratio
- relatively inexpensive
  - Portland cement
  - aggregate
  - water
Concrete Materials

- reinforcement
  - deformed bars
  - prestressing strand
  - stirrups
  - development length
  - anchorage
  - splices

Concrete Materials

- fire resistance
  - most fire-resistant structural material
  - low rate of penetration
  - retains strength if exposure not too long
    - stable to 900 – 1200 °F internally
    - loses 50% after that
  - no toxic fumes
  - cover necessary to protect steel

Concrete Beams

- types
  - reinforced
  - precast
  - prestressed

Concrete Beams

- deformation
  - camber (elastic)
    - hogging ↑
    - sagging ↓
  - shrinkage strain
    - 200-400 x 10^-6
    - about 2-3 years
  - creep strain
    - 2~3 times elastic strain
    - about 2-3 years
Concrete Beams

- shear
  - vertical
  - horizontal
  - combination:
    - tensile stresses at 45°
- bearing
  - crushing

Concrete Beam Design

- composite of concrete and steel
- American Concrete Institute (ACI)
  - design for failure
  - strength design (LRFD)
    - service loads x load factors
    - concrete holds no tension
    - failure criteria is yield of reinforcement
    - failure capacity x reduction factor
    - factored loads < reduced capacity
  - concrete strength = $f'_c$

Behavior of Composite Members

- plane sections remain plane
- stress distribution changes

Transformation of Material

- $n$ is the ratio of $E$'s
  \[ n = \frac{E_2}{E_1} \]
- effectively widens a material to get same stress distribution
Stresses in Composite Section

- with a section transformed to one material, new I
- stresses in that material are determined as usual
- stresses in the other material need to be adjusted by n

\[ n = \frac{E_2}{E_1} = \frac{E_{\text{steel}}}{E_{\text{concrete}}} \]

\[ f_c = -\frac{M_y}{I_{\text{transformed}}} \]

\[ f_s = -\frac{M_y n}{I_{\text{transformed}}} \]

Reinforced Concrete Analysis

- for stress calculations
  - steel is transformed to concrete
  - concrete is in compression above n.a. and represented by an equivalent stress block
  - concrete takes no tension
  - steel takes tension
  - force ductile failure

Location of n.a.

- ignore concrete below n.a.
- transform steel
- same area moments, solve for x

\[ bx \cdot \frac{x}{2} - nA_s (d - x) = 0 \]
T sections

- n.a. equation is different if n.a. below flange

\[ b_f h_f \left( x - \frac{h_f}{2} \right) + b_w \left( x - h_f \right) \frac{x - h_f}{2} - nA_s (d - x) = 0 \]

ACI Load Combinations

- 1.4D
- 1.2D + 1.6L + 0.5(L<sub>r</sub> or S or R)
- 1.2D + 1.6(L<sub>r</sub> or S or R) + (1.0L or 0.5W)
- 1.2D + 1.0W + 1.0L + 0.5(L<sub>r</sub> or S or R)
- 1.2D + 1.0E + 1.0L + 0.2S
- 0.9D + 1.0W
- 0.9D + 1.0E

*can also use old ACI factors

Reinforcement

- deformed steel bars (rebar)
  - Grade 40, \( F_y = 40 \text{ ksi} \)
  - Grade 60, \( F_y = 60 \text{ ksi} \) - most common
  - Grade 75, \( F_y = 75 \text{ ksi} \)
  - US customary in # of 1/8” \( \phi \)

- longitudinally placed
  - bottom
  - top for compression reinforcement
  - spliced, hooked, terminated...

Reinforced Concrete Design

- stress distribution in bending

Wang & Salmon, Chapter 3
**Force Equations**

- $C = 0.85 f'_c b a$
- $T = A_s f_y$

where
- $f'_c$ = concrete compressive strength
- $a$ = height of stress block
- $\beta_1$ = factor based on $f'_c$
- $c$ = location to the n.a.
- $b$ = width of stress block
- $f_y$ = steel yield strength
- $A_s$ = area of steel reinforcement

**Equilibrium**

- $T = C$
- $M_n = T(d-a/2)$
- $d$ = depth to the steel n.a.
- $\phi$ = steel ratio
  - $\phi = 0.65 + (\epsilon_s - \epsilon_y) \frac{0.25}{(0.005 - \epsilon_y)} \geq 0.65$
  - $M_u \leq \phi M_n \quad \phi = 0.9$ for flexure*
  - $\phi M_n = \phi T(d-a/2) = \phi A_s f_y (d-a/2)$

**Over and Under-reinforcement**

- over-reinforced
  - steel won’t yield
- under-reinforced
  - steel will yield
- reinforcement ratio
  - $\rho = \frac{A_s}{bd}$
  - use as a design estimate to find $A_s$, $b$, $d$
  - max $\rho$ is found with $\epsilon_{steel} \geq 0.004$ (not $\rho_{bal}$)
  - *with $\epsilon_{steel} \geq 0.005$, $\phi = 0.9$

**$A_s$ for a given Section**

- several methods
  - guess $a$ and iterate
    1. guess $a$ (less than n.a.)
    2. $A_s = \frac{0.85 f'_c b a}{f_y}$
    3. solve for $a$ from $M_u = \phi A_s f_y (d-a/2)$
    4. repeat from 2. until a from 3. matches a in 2.
**A_s For Given Section (cont)**

- **chart method**
  - Wang & Salmon
  - Fig. 3.8.1 \( R_n \) vs. \( \rho \)
    1. calculate \( R_n = \frac{M_n}{bd^2} \)
    2. find curve for \( f_c' \) and \( f_y \) to get \( \rho \)
    3. calculate \( A_s \) and \( a \)
- simplify by setting \( h = 1.1d \)

**ACI Shear Values**

- \( V_u \) is at distance \( d \) from face of support
- shear capacity: \( V_c = \nu_c \times b_w d \)
  - where \( b_w \) means thickness of web at n.a.
- shear stress (beams)
  - \( \nu_c = 2 \lambda \sqrt{f_c'} \)
  - \( \phi = 0.75 \) for shear
  - \( \phi V_c = \phi 2 \lambda \sqrt{f_c'} b_w d \)
  - \( \lambda \) for lightweight materials
- shear strength: \( V_u \leq \phi V_c + \phi V_s \)
  - \( V_s \) is strength from stirrup reinforcement

**Shear in Concrete Beams**

- flexure combines with shear to form diagonal cracks
- horizontal reinforcement doesn’t help
- stirrups = vertical reinforcement

**ACI Shear Values**

- shear capacity:
  \[ V_s = \frac{A_v f_y d}{s} \]
  - \( A_v \) = area in all legs of stirrups
  - \( s \) = spacing of stirrup
- may need stirrups when concrete has enough strength!
Required Stirrup Reinforcement

- spacing limits

<table>
<thead>
<tr>
<th>Table 3-8 ACI Provisions for Shear Design*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{bc}$ vs $V_{bc}^{*}$</td>
</tr>
<tr>
<td>Required area of stirrups, $A_{sh}$**</td>
</tr>
<tr>
<td>greater of $0.5 b d$ and $0.75 f_{y} b d$</td>
</tr>
<tr>
<td>$V_{bc} &gt; V_{bc}^{*}$</td>
</tr>
</tbody>
</table>

*Members subjected to shear and tension only, $V_{bc}' = 0.012 f_{y} b d$ (ACI 11.3.1.1)
**$A_{sh} = 2.6 A_{sh}$ for $1$ stirrup, $1.5 A_{sh}$ (ACI 11.3.1.1)

Recommended Limit

- $V_{bc} = V_{bc}^{*}$

Maximum

- $V_{bc} < V_{bc}^{*}$

Deflection Limits

- relate to whether or not beam supports or is attached to a damageable non-structural element

- need to check service live load and long term deflection against these

| L/180 | roof systems (typical) – live |
| L/240 | floor systems (typical) – live + long term |
| L/360 | supporting plaster – live |
| L/480 | supporting masonry – live + long term |

Concrete Deflections

- elastic range
  - $E_{c}$ (with $f'_{c}$ in psi)
    - normal weight concrete (~ 145 lb/ft³)
      $E_{c} = 57,000 f'_{c}$
    - concrete between 90 and 155 lb/ft³
      $E_{c} = \sqrt[3]{0.75 f'_{c}}$

- cracked
  - $I$ cracked
  - $E$ adjusted

Prestressed Concrete

- impose a longitudinal force on a member in order to withstand more loading until the member reaches a tensile limit
Prestressed Concrete

- pretensioned
  - reinforcement bonded
- post-tensioned
  - bonded or unbonded
  - end bearing
- precast
  - concrete premade in a position other than its final position in the structure

Prestressed Concrete

- high strength tendons
  - grade 250
  - grade 270

Prestressed Concrete

- axial prestress \((e=0)\)

\[
f'_{t} = \frac{P}{A} - \frac{M_{c}}{I_{g}}
\]

\[
f'_{b} = \frac{P}{A} + \frac{M_{c}}{I_{g}}
\]

- axial prestress \((e\neq 0)\)

\[
f'_{t} = \frac{P}{A} + \frac{P_{ec}}{I_{g}} - \frac{M_{c}}{I_{g}} = \frac{P}{A} \left(1 - \frac{ec_{0}}{r}ight) - \frac{M_{c}}{I_{g}}
\]

\[
f'_{b} = \frac{P}{A} + \frac{P_{ec}}{I_{g}} + \frac{M_{c}}{I_{g}} = \frac{P}{A} \left(1 + \frac{ec_{0}}{r}ight) + \frac{M_{c}}{I_{g}}
\]

(remember \(r = \frac{T_{1}}{A}\))
Prestressed Concrete

- Self weight
- Design load
- Reinforced Concrete Construction 38 Lecture 9
- Architectural Structures III
- ARCH 631
- F2007abn

Composite Beams

- concrete
  - in compression
- steel
  - in tension
- shear studs

Continuous Beams

- reduced size
- reduced moments
- moments can reverse with loading patterns
- need top & bottom reinforcement
- sensitive to settlement
Approximate Depths

Concrete Columns

- columns require
  - ties or spiral reinforcement to “confine” concrete (#3 bars minimum)
  - minimum amount of longitudinal steel (4 bars minimum)

Concrete Columns

- effective length in monolithic casts must be found with respect to stiffness of joint
- not slender when
  \[ \frac{kL}{r} < 22 \]

Concrete Columns

- not braced
Concrete Columns

- $P_o$ – no bending
  \[ P_o = 0.85 f'_c (A_g - A_{st}) + f_y A_{st} \]

- $\phi_c = 0.65$ for ties with $P_n = 0.8P_o$
- $\phi_c = 0.70$ for spirals
  with $P_n = 0.85P_o$
- $P_u \leq \phi_c P_n$
- nominal axial capacity:
  - presumes steel yields
  - concrete at ultimate stress

Columns with Bending

- eccentric loads can cause moments
- moments can change shape and induce more deflection ($P - \Delta$)

Columns with Bending

- for ultimate strength behavior, ultimate strains can’t be exceeded
  - concrete 0.003
  - steel $\frac{f_y}{E_s}$
- $P$ reduces with $M$
Concrete Floor Systems

• types & spanning direction

Concrete Floor Systems

• flexure design as T-beams (+/- M)
• increase of 10% $V_c$ permitted
• one-way and two-way moments
• slabs need steel
• effective width is
  – $L/4$
  – $b_w + 16t$
  – center-to-center of beams

One-way Joists

– standard stems
– 2.5” to 4.5” slab
– ~30” widths
– reusable forms
One-way

Joists
– wide pans
– 5’, 6’ up
– light loads & long spans
– one-leg stirrups

Two-way

Joists
– domed pans
– 3’, 4’ & 5’

Construction Supervision

• proper placement of all reinforcement
  – welding
  – splices

• mix design
  – slump
  – in-situ strength
    • cast cylinders
    • cylinder cores – if needed