structural analysis
(statics & mechanics)

Structure Requirements

• strength & equilibrium
  – safety
  – stresses not greater than strength
  – adequate foundation

Figure 1.16 Equilibrium and Stability?—sculpture by Richard Eyer. Photo by author.

Structure Requirements

• stability & stiffness
  – stability of components
  – minimum deflection and vibration
  – adequate foundation

Figure 1.15 Stability and strength of a structure—the collapse of a portion of the UW Husky stadium during construction (1987) due to a lack of adequate bracing to ensure stability. Photo by author.
Structure Requirements

- economy and construction
  - minimum material
  - standard sized members
  - simple connections and details
  - maintenance
  - fabrication/erection

Relation to Architecture

“The geometry and arrangement of the load-bearing members, the use of materials, and the crafting of joints all represent opportunities for buildings to express themselves. The best buildings are not designed by architects who after resolving the formal and spatial issues, simply ask the structural engineer to make sure it doesn’t fall down.” - Onouy & Kane

Structural Loads - STATIC

- dead load
  - static, fixed, includes material weights, fixed equipment
- live load
  - transient and moving loads (including occupants)
- snow load

Structural Loads – STATIC & DYNAMIC

- wind loads
  - dynamic, wind pressures treated as lateral static loads on walls, pressure or suction
  - pressure determined from wind velocity, \( q_h \)
  - dynamic effects include motion from buffeting or “vortex shedding”

\[
F_W = C_d q_h A
\]
**Structural Loads - DYNAMIC**

- **earthquake loads**
  - seismic, movement of ground (3D)
  - building mass responds
  - static models often used, \( V \) is static shear

- **impact loads**
  - rapid, energy loads

\[
V = \frac{ZICW}{R_W}
\]

**Dynamic Response**

- **period of vibration or frequency**
  - wave
  - sway/time period

- **damping**
  - reduction in sway

- **resonance**
  - amplification of sway

**Statics & Mechanics Review**

- **how loads affect our structures**
  - **statics**: things don’t move
    - forces
    - supports & connections
    - equilibrium
  - **mechanics**: things can change shape
    - stress & strain
    - deflections
    - buckling
**Structural Math**

- quantify environmental loads
  - how big is it?
- evaluate geometry and angles
  - where is it?
  - what is the scale?
  - what is the size in a particular direction?
- quantify what happens in the structure
  - how big are the internal forces?
  - how big should the beam be?

**Physical Math**

- physics takes observable phenomena and relates the measurement with rules: mathematical relationships
- need
  - reference frame
  - measure of length, mass, time, direction, velocity, acceleration, work, heat, electricity, light
  - calculations & geometry

**Units**

- measures
  - US customary & SI

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<tr>
<th>Units</th>
<th>US</th>
<th>SI</th>
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**Vectors**

- scalars – any quantity
- vectors - quantities with direction
  - like displacements
  - summation results in the “straight line path” from start to end
  - normal vector is perpendicular to something
**Forces & Reactions**

- Newton’s 3\textsuperscript{rd} law:
  - for every force of action there is an equal and opposite reaction along the same line.
- External forces act on bodies
  - can cause moments.
- Internal forces are
  - in bodies
  - between bodies (connections).

**Force Components**

- convenient to resolve into 2 vectors
- at right angles
- in a “nice” coordinate system
- $\theta$ is between $F_x$ and $F$ from $F_x$.

$$F_x = F \cos \theta$$
$$F_y = F \sin \theta$$
$$F = \sqrt{F_x^2 + F_y^2}$$
$$\tan \theta = \frac{F_y}{F_x}$$

**Load Types**

- Weight ($F = ma$)
  - $W = \gamma A$
- Concentrated
- Distributed
  - Uniform
  - Linear
- Friction
  - $F = \mu N$

**Load Tracing**

- Tributary load
  - think of water flow
  - “concentrates” load of area into center

$$w = \left(\frac{\text{load}}{\text{area}}\right) \times \text{(tributary width)}$$
Moments
- defined by magnitude and direction
- units: N·m, k·ft
- direction:
  + ccw (right hand rule)
  - cw
- value found from \( F \) and \( \perp \) distance
  \[ M = F \cdot d \]
- \( d \) also called “lever” or “moment” arm

Equilibrium
- analytically
  \[ R_x = \sum F_x = 0 \]
  \[ R_y = \sum F_y = 0 \]
  \[ M = \sum M = 0 \]
- free body diagrams

Free Body Diagram
- FBD (sketch)
- tool to see all forces on a body or a point including
  - external forces
  - weights
  - force reactions
  - external moments
  - moment reactions
  - internal forces

Supports and Connections
- Roller
- Rocker
Supports and Connections

Centroid
- “average” x & y of an area
- for a volume of constant thickness
  - $\Delta W = \gamma \Delta A$  where $\gamma$ is weight/volume
  - center of gravity = centroid of area

$$\bar{x} = \frac{\sum(x\Delta A)}{A}$$
$$\bar{y} = \frac{\sum(y\Delta A)}{A}$$

Moments of Inertia
- 2nd moment area
  - math concept
  - area x (distance)$^2$
- need for behavior of
  - beams
  - columns

$$I_x = I_{cx} + Ad_y^2$$

Internal and Pin Forces
- 3 equations per three-force body
- two-force body forces in line
- 2 reactions per pin + support forces
**Internal Beam V & M (+P)**

- maximums needed for design
- $M_{\text{max}}$ at $V = 0$

![Internal Beam V & M (+P)](image)

**Deflected Shape**

- positive bending moment
  - tension in bottom, compression in top
- negative bending moment
  - tension in top, compression in bottom
- zero bending moment
  - inflection point

![Deflected Shape](image)

**Stress**

- stress is a term for the intensity of a force, like a pressure
- internal or applied
- force per unit area

$$\text{stress} = \frac{P}{A}$$

**Stress Types**

- normal stress is normal to the cross section
  $$f_{\text{t or c}} = \frac{P}{A}$$
- shear stress parallel to a surface
  $$f_v = \frac{P}{A} = \frac{P}{td}$$
Stress Types

- **bearing stress on a surface by contact in compression**
  \[ f_p = \frac{P}{A} = \frac{P}{td} \]
- **torsional stress by shear from twisting**
  \[ f_v = \frac{Tp}{J} \]

Bolt Stresses

- **single shear**
  \[ f_v = \frac{P}{A} = \frac{P}{\pi d^2/4} \]
- **double shear**
  \[ f_v = \frac{P}{2A} = \frac{P}{\pi d^2/4} \]
- **bearing**
  \[ f_p = \frac{P}{A_{projected}} = \frac{P}{td} \]

Connections Resisting Shear

- **plates with**
  - nails
  - rivets
  - bolts
- **splices**
  \[ \frac{V_{longitudinal}}{p} = \frac{VQ}{I} \]
  \[ nF_{connector} \geq \frac{VQ_{connected area}}{I} \cdot p \]

Bending Stresses

- **tension and compressive stress caused by bending**
  \[ f_b = \frac{Mc}{I} = \frac{M}{S} \]
- **shear stress from bending**
  \[ f_v = \frac{VQ}{Ib} \]
Strain
- materials deform
- axially loaded materials change length
- bending materials deflect

\[ \text{strain} = \frac{\Delta L}{L} \]

Problem Solving
1. STATICS: equilibrium of external forces, internal forces, stresses
2. GEOMETRY: cross section properties, deformations and conditions of geometric fit, strains
3. MATERIAL PROPERTIES: stress-strain relationship for each material obtained from testing

Stress to Strain
- important to us in \( f - \varepsilon \) diagrams:
  - straight section
  - LINEAR-ELASTIC \( f = E \cdot \varepsilon \)
  - recovers shape (no permanent deformation)

\[ \delta = \frac{PL}{AE} \]

Behavior Types
- brittle
- semi-brittle
Plastic Behavior

- ductile

![Stress-strain diagram](image)

- at yield stress

Thermal Deformation

- $\alpha$ - the rate of strain per degree
- UNITS: $/\circ F$, $/\circ C$
- length change: $\delta_T = \alpha(\Delta T)L$
- thermal strain: $\varepsilon_T = \alpha(\Delta T)$
  - no stress when movement allowed

Maximum Stresses

- if we need to know where $\max f$ and $f_v$ happen:

\[
\begin{align*}
\theta &= 0^\circ \rightarrow \cos \theta = 1 \quad f_{\text{max}} = \frac{P}{A_o} \\
\theta &= 45^\circ \rightarrow \cos \theta = \sin \theta = \sqrt{0.5} \\
f_v\text{-max} &= \frac{P}{2A_o} = \frac{f_{\text{max}}}{2}
\end{align*}
\]

Beam Deflections

- curvature, $R$

\[
R = \frac{M}{EI} \quad \text{curvature} = \frac{M(x)}{EI}
\]

\[
\begin{align*}
\theta &= \text{slope} = \int \frac{M(x)}{EI} \, dx \\
\Delta &= \text{deflection} = \int \int \frac{M(x)}{EI} \, dx \\
y_{\text{max}}(x) &= \Delta_{\text{actual}} \leq \Delta_{\text{allowable}}
\end{align*}
\]
Column Stability

- short columns
  \[ f_{\text{critical}} = \frac{P_{\text{actual}}}{A} < F_a \]
- slenderness ratio \( r = L_e / r \) (L/d)
- radius of gyration \( r = \sqrt{I / A} \)

\[ f_{\text{critical}} = \frac{P_{\text{critical}}}{A} = \left( \frac{\pi^2 E A r^2}{A(L_e)^2} \right) = \left( \frac{\pi^2 E}{L_e / r} \right)^2 \]

\[ P_{\text{critical}} = \frac{\pi^2 E A}{(L_e / r)^2} \]

Column Stresses

- when a column gets stubby, \( F_y \) will limit the load
- real world has loads with eccentricity
- end conditions \( L_e = K \cdot L \)