ARCH 631. Topic 4 Reading Notes

(includes Appendices 3-11)

- Beams are simple and common to construct, but have the more complex load-carrying actions than trusses or cables
- Beams commonly are used in a hierarchical arrangement with planar surfaces of decking or planks supported at intervals by members (joists) with larger spans which can be supported by other members (girders) and the loads to each increase progressively; three levels are typically the maximum; one and two levels are common
- Increase in length does not give a direct proportional increase in depth
- Stresses are a function of distribution of cross section area; deeper tends to be better
- Primary design variables are magnitudes of the loads, distances between the loads, the support conditions (important); end restraints make the beam stiffer
- Vertical members can behave as beams because they are bending; curved members may also bend and are designed as beams
- Internal forces in beams are primarily bending moments and shear forces which ensure equilibrium; generated through shear and bending stresses
- Under bending, the deformations horizontally vary in a linear way with maximum elongation on the faces; the layer with no deformation is the neutral axis and corresponds to the centroid
- For linear-elastic materials, the strains result in linear distribution of bending stress from the neutral axis
- The tensile and compressive stresses result in the internal bending moment
- Bending stress is a function of the vertical distance and cross section shape: $f = My/I$
- Shear stresses within the cross section result in the internal shear, while there are also shear stresses acting horizontally
- Besides considering stresses from bending and shear, beam design must consider bearing stress, torsional stress, combined stress, the shear center of the cross section, principal stress, and deflection
- Bending stress doesn't depend on the material
- Little f refers to actual stress, while big F generally refers to a limit or allowable stress which depends on material
- With an increase in moment and distance from the neutral axis, the bending stress increases
- With an increase in moment of inertia, the bending stress decreases
- The maximum bending moment occurs at the outer fibers of the beam, distance of c
- For a rectangular cross section, $f = M/(bh^2/6)$ or M/S where $S = bh^2/6$
- For design, the maximum bending stress occurs at the maximum moment and cannot exceed the design limit or it is termed overstressed and unacceptable
- Doubling the depth decreases bending stress by a factor of 4 (depth is squared in the denominator)
- I (moment of inertia) is defined as integral over the area of y^2 (second moment of an area)

- Centroid location is defined by the integral of the average area center over the area = 0 (which means the first moment of the area above the centroid equals the first moment are of the area below the centroid)
- Centroid is obvious in a symmetrical shape
- The Parallel-axis Theorem is used to determine moment of inertia for a shape with holes or built-up from basic shapes
- When a shape is not symmetrical top-to-bottom, the maximum stress occurs at the furthest distance away from the neutral axis
- A composite is a member mate of multiple materials that work together to carry loads (like reinforced concrete)
- For allowable stress design, the section modulus required is a function of the maximum moment and the allowable bending stress for the material; $S_{required} = M/F_b$; where S = I/c
- If a beam with varying bending moment is to have a constant cross section, the maximum bending moment must be used for design; but depths can be tapered for a well=defined bending moment distribution
- Lateral buckling of a beam is an instability duce to compressive stresses in the upper region of the beam (for positive bending) and can be prevented by making the beam stiff in the lateral direction or by transverse bracing
- The horizontal planes of a beam have a tendency to slide such that horizontal shear stresses are developed which are a result of unequal bending stresses along the length of the beam
- Shear stress is zero at the free surfaces and increases at the neutral axis for a constant cross section from the expression $f_v = VQ/Ib$ (Q is first moment arm, b is width)
- Shear stress for a rectangular beam $f_v = (3/2V)/A$
- Shear stress for a wide-flange beam $f_v = V/A_{web} = V/td$
- Bearing stress is compressive stress developed at the contact area between two loaded members; common at supports $f_p = P/A$
- Torsion stress results from twisting moment or torque $\tau = Mr/J$ (circular shape) which looks like $f = Mc/I$; specific equations are given for rectangular shapes, thin-walled tubes that are open or closed
- Vertically unsymmetrical shapes will twist under bending and shear through the shear center found from geometry and the shear flow; loading through the shear center won't twist
- For a simply supported beam, increasing w or L increases the deflection; increasing I or E decreases the deflection
- Deflection equations have L raised to some power, so deflections are very sensitive to length
- Deflections can be reduced by constraining the supports
- The limit for beam deflection is not always obvious, but should not interfere with other building elements; common to see limits as L/#
- The interaction of shear stresses and bending stresses result in resultant of principal stresses (maximum tensile and compressive stresses at other angles from the axis)

- Stress trajectories are lines showing the direction of principal stress
- Finite element methods allow for investigation of stresses of complex models by discretizing the shape into small elements and enforcing stress relationships and geometric constraints
- Beam design depends on methodology but finds the size and shape based on maximum internal forces and acceptable deflections
- Maximizing I with minimizing area results in the wide-flange shape (flanges resist bending while the web resists shear stresses)
- Beam design can also vary the material properties, like the strength of the laminates in glu-lam
- Beam design can also vary the depth to correspond with the bending moment diagram (considered efficient to reduce material where not necessary)
- Moving support locations or changing boundary conditions reduce the magnitudes of the bending moments or change their distribution; overhangs on one or both ends do this by changing some of the positive moment into negative moment – section size can be smaller
- Positions of overhang supports common referred to as the 1/5 point (both ends) or 1/3 point (one end)
- Statically determinate beam reactions/forces are independent of the cross section or material
- Statically indeterminate beams (constrained or continuous) are dependent on the cross section and material for the sizes of the reactions and forces; tend to be more rigid and have smaller moments
- Disadvantage of indeterminate beams is sensitivity to support settlements and thermal effects which cause redistribution of maximum stresses
- Effective length is the equivalent span of a simply supported beam (between 0 moments / points of inflection)
- Moment distribution method was popular hand calculation method to determine moments in a statically determinate beam; now we have computer methods (matrix analysis or finite element)
- Approximate analysis methods rely on generally locating the points of inflection and solving statically knowing there is zero bending moment there
- MULTIFRAME, like other programs, requires the material and cross section to analyze a statically indeterminate structure
- Changing the beam stiffness (EI) over the length has the effect of changing the location of the points of inflection
- Support settlements can induce curvature in a beam, resulting in changing the bending stresses (usually increasing them)
- Cable supported beams have a system of compression struts and cables; the positive moments increase while the negative moments over the intermediate supports decreases; deeper struts generally make for a stiffer support condition
- Partial-loading conditions can result in an increase in the maximum positive moment in a continuous span from a continuously loaded member; the maximum negative moment changes too
- Critical loading conditions are those that produce the maximum deflection and maximum values for design

Updated material

- Moments of distributed loads can be determined from the total "area" of the distributed load (sum of (*w·dx)* by the lever arms or $M_o = \int w(dx)$)
- Centroid is the center of area of a geometric form (as compared to center of mass in a body or center of gravity when there is acceleration due to mass attraction); this location will have a balanced area either side (left/right or top/bottom) of the centroidal axes
- The coordinates are defined as $\bar{x} = \frac{\int_A xdA}{A}$ and $\bar{y} = \frac{\int_A ydA}{A}$; with composite areas (comprised of simpler parts) these definitions are expressed as $\bar{x} = \frac{\Sigma Ax_i}{A}$ and $\bar{y} = \frac{\Sigma Ay_i}{A}$. (*Note: the x_i and y_i are the wrong typeset in the text on page 518.)*
- The first moment areas are $x{\cdot}dA$ and $y{\cdot}dA$, and when taken about the centroid axes, $\int_{A_c} xdA = 0$ and $\int_{A_c} ydA = 0$; the first moment area can be negative with negative areas (or negative distances)
- For a symmetrical area, the centroid coincides with the axis of symmetry
- Moment of inertia (also known as second moment area) is a common expression important to analyzing beams, columns and other elements; it is the sum of the products of the areas by the square of the distance from the axis represented as $I_x = \int_A y^2 dA$ and $I_y = \int_A x^2 dA$; the moment of inertia will *always* be positive
- The parallel axis theorem allows the moment of inertia to be found about a parallel axis and is useful for composite areas: $I = \Sigma I + \Sigma Ad^2$; when all the areas in the composite have their centroid axis on the composite centroid axis all the d's = 0 and $I_c = \Sigma I$
- Centroids and moment of inertia for composite shapes having negative areas use negative A and I
- Bending stress can be determined from the strain of a cross section plane and the linear relationship to stress in addition to force equilibrium ($\Sigma F_x = \int_A f_y dA = 0$) , proving that the location of zero stress (neutral axis) is at the centroid, and that the stress increases linearly from the neutral axis to the extreme fiber in bending, *c*
- The sum of the moments from the stress on a cross section must equal the bending moment, deriving the stress relationship with respect to the moment of inertia as $f_y = \frac{My}{I}$ and $f_{b-\max} = \frac{Mc}{I}$
- Beam horizontal shear stresses result from the difference in total horizontal force due to bending stress on either side of a "infinitesimal element" ($dH = f_h(b\,dx) = F_{2-from\,M_2} - F_{1-from\,M_1}$) and because the bending stresses are functions of the moment and the moment of inertia, the shear stress is found to be $f_v = \frac{VQ}{Ib}$; the maximum occurs where Q is the largest which is the neutral axis (*as long as b isn't the smallest with a bigger Q/b ratio somewhere else –abn)*; shear is parabolically distributed across a rectangular cross section

- Bending moment is related to the deflection in a beam by the strain relation of a curve with a "radius of curvature" - ρ (more commonly named "R") through the elongation of an arc length, *y dθ,* resulting in $1/\rho = M/EI$; with M=0 the curvature goes to infinity and the beam will be flat; with M very big, the radius is small and it will be more sharply curved ("more curvature")
- Deflection is related to the radius of curvature, and subsequently to moment through $M = \left(\frac{d^2y}{dx^2}\right)EI$; double integration produces *y* when the constants of integration are found knowing where deflections and slopes are restricted (or 0): $y = \iint \frac{M}{EI}dx$
- Moment-area theorems can be used to calculate either the slope or the deflection of any point on a beam if the moment diagram is known from change in slope- $\theta_{B/A} = \int_A^B \frac{M}{EI}dx$ and change in tangent to the slope- $t_{A/B} = \int_A^B x\frac{M}{EI}dx$ where *x* is the distance from A to the centroid of the area
- Double integration of $y = \iint \frac{M}{EI}dx$ can be used to find moment reactions if you know the equation for the moment curve for a statically determinate beam
- The deflection method *(probably the force-displacement method*) determines the unknowns in the equations as the displacements using a statically determinate beam with superimposed reaction forces and moments, and enforces the actual displacements at the supports