Concrete
- columns
- beams
- slabs
- domes
- footings

Concrete Construction
- cast-in-place
- tilt-up
- prestressing
- post-tensioning

Concrete Materials
- low strength to weight ratio
- relatively inexpensive
  - Portland cement
  - aggregate
  - water
Concrete Materials

- reinforcement
  - deformed bars
  - prestressing strand
  - stirrups
  - development length
  - anchorage
  - splices

Concrete Materials

- fire resistance
  - most fire-resistive structural material
  - low rate of penetration
  - retains strength if exposure not too long
    - stable to 900 – 1200 °F internally
    - loses 50% after that
  - no toxic fumes
  - cover necessary to protect steel

Concrete Beams

- types
  - reinforced
  - precast
  - prestressed

Concrete Beams

- deformation
  - camber (elastic)
    - hogging
    - sagging
  - shrinkage strain
    - 200-400 x 10^{-6}
    - about 2-3 years
  - creep strain
    - 2~3 times elastic strain
    - about 2-3 years
Concrete Beams

- shear
  - vertical
  - horizontal
  - combination:
    - tensile stresses at 45°
- bearing
  - crushing

Concrete Beam Design

- composite of concrete and steel
- American Concrete Institute (ACI)
  - design for failure
  - strength design (LRFD)
    - service loads x load factors
    - concrete holds no tension
    - failure criteria is yield of reinforcement
    - failure capacity x reduction factor
    - factored loads < reduced capacity
  - concrete strength = f'_c

Behavior of Composite Members

- plane sections remain plane
- stress distribution changes

Transformation of Material

- n is the ratio of E’s
  \[ n = \frac{E_2}{E_1} \]
- effectively widens a material to get same stress distribution

\[
f_1 = E_1 \varepsilon = -\frac{E_1 y}{\rho} \quad f_2 = E_2 \varepsilon = -\frac{E_2 y}{\rho}
\]
Stresses in Composite Section

- with a section transformed to one material, new I
  - stresses in that material are determined as usual
  - stresses in the other material need to be adjusted by n

\[ n = \frac{E_2}{E_1} = \frac{E_{\text{steel}}}{E_{\text{concrete}}} \]

\[ f_c = -\frac{M_y}{I_{\text{transformed}}} \]

\[ f_s = -\frac{M_y}{I_{\text{transformed}}} \]

Reinforced Concrete Analysis

- for stress calculations
  - steel is transformed to concrete
  - concrete is in compression above n.a. and represented by an equivalent stress block
  - concrete takes no tension
  - steel takes tension
  - force ductile failure

Reinforced Concrete - stress/strain

Location of n.a.

- ignore concrete below n.a.
- transform steel
- same area moments, solve for x

\[ bx \cdot \frac{x}{2} - nA_s (d - x) = 0 \]
T sections

- n.a. equation is different if n.a. below flange

\[ \begin{align*}
  b_y h_f \left( x - \frac{h_f}{2} \right) + (x - h_f) b_w \left( x - \frac{h_f}{2} \right) - n A_s (d - x) &= 0
\end{align*} \]

ACI Load Combinations*

- 1.4D
- 1.2D + 1.6L + 0.5(L_r or S or R)
- 1.2D + 1.6(L_r or S or R) + (1.0L or 0.5W)
- 1.2D + 1.0W + 1.0L + 0.5(L_r or S or R)
- 1.2D + 1.0E + 1.0L + 0.2S
- 0.9D + 1.0W
- 0.9D + 1.0E

*can also use old ACI factors

Reinforcement

- deformed steel bars (rebar)
  - Grade 40, \( F_y = 40 \text{ ksi} \)
  - Grade 60, \( F_y = 60 \text{ ksi} \) - most common
  - Grade 75, \( F_y = 75 \text{ ksi} \)
  - US customary in # of 1/8" \( \phi \)
- longitudinally placed
  - bottom
  - top for compression reinforcement
  - spliced, hooked, terminated...

Reinforced Concrete Design

- stress distribution in bending

\[ \begin{align*}
  \text{actual stress} &= \frac{0.85 f'_c}{\beta_{sl} a/2}
\end{align*} \]
Force Equations

- \( C = 0.85 f'_c ba \)
- \( T = A_s f_y \)
- where
  - \( f'_c = \) concrete compressive strength
  - \( a = \) height of stress block
  - \( b = \) width of stress block
  - \( f_y = \) steel yield strength
  - \( A_s = \) area of steel reinforcement

Equilibrium

- \( T = C \)
- \( M_n = T(d-a/2) \)
  - \( d = \) depth to the steel n.a.
- with \( A_s \)
  - \( a = A_s f_y \)
  - \( M_u \leq \phi M_n \)
    - \( \phi = 0.9 \) for flexure
    - \( \phi M_n = \phi T(d-a/2) = \phi A_s f_y (d-a/2) \)

Over and Under-reinforcement

- over-reinforced
  - steel won’t yield
- under-reinforced
  - steel will yield
- reinforcement ratio
  - \( \rho = \frac{A_s}{bd} \)
  - use as a design estimate to find \( A_s, b, d \)
  - max \( \rho \) is found with \( \epsilon_{\text{steel}} \geq 0.004 \) (not \( \rho_{\text{bal}} \))

A_s for a given Section

- several methods
  - guess \( a \) and iterate
    1. guess \( a \) (less than n.a.)
    2. \( A_s = \frac{0.85 f'_c b a}{f_y} \)
    3. solve for \( a \) from \( M_u = \phi A_s f_y \left( d - \frac{a}{2} \right) \)
      \[ a = 2 \left( d - \frac{M_u}{\phi A_s f_y} \right) \]
    4. repeat from 2. until \( a \) from 3. matches \( a \) in 2.
**A_s For Given Section (cont)**

- chart method
  - Wang & Salmon
    - Fig. 3.8.1  \( R_n \) vs. \( \rho \)
    1. calculate \( R_n = \frac{M_n}{bd^2} \)
    2. find curve for \( f'_c \) and \( f_y \) to get \( \rho \)
    3. calculate \( A_s \) and \( a \)
- simplify by setting \( h = 1.1d \)

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**Shear in Concrete Beams**

- flexure combines with shear to form diagonal cracks

- horizontal reinforcement doesn’t help
- stirrups = vertical reinforcement

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**ACI Shear Values**

- \( V_u \) is at distance \( d \) from face of support
- shear capacity: \( V_c = \nu_c \times b_w d \)
  - where \( b_w \) means thickness of web at n.a.
- shear stress (beams)
  - \( \nu_c = 2\sqrt{f'_c} \)
  - \( \phi = 0.75 \) for shear
  - \( \phi V_c = \phi 2\sqrt{f'_c} b_w d \)
  - \( f'_c \) is in psi
- shear strength: \( V_u \leq \phi V_c + \phi V_s \)
  - \( V_s \) is strength from stirrup reinforcement

**Stirrup Reinforcement**

- shear capacity:
  \[ V_s = \frac{A_v f_y d}{s} \]
  - \( A_v \) = area in all legs of stirrups
  - \( s \) = spacing of stirrup
- may need stirrups when concrete has enough strength!
Required Stirrup Reinforcement

- spacing limits

<table>
<thead>
<tr>
<th>Table 3-8 ACI Provisions for Shear Design*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_c &lt; \frac{f_y}{2}$</td>
</tr>
<tr>
<td>$A_y$**</td>
</tr>
<tr>
<td>Required</td>
</tr>
<tr>
<td>Recommended Minimum†</td>
</tr>
<tr>
<td>Maximum‡ (ACI 11.5.4)</td>
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<td></td>
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</tbody>
</table>

*Members subjected to shear and flexure only. $V_c = \frac{f_y}{2} \sqrt{b_d d}$, $d = 0.75$ (ACI 11.5.1.1)
**$A_y = 2 \times A_y$ for 8-stirrup; $I_y \leq 60 \text{ksi}$ (ACI 11.5.2)
†A practical limit for minimum spacing is 4/4
‡Maximum spacing based on minimum stirrup reinforcement ($\geq \frac{A_y}{50\text{lb}}$) must also be considered (ACI 11.5.3).

Concrete Deflections

- elastic range
  - I transformed
  - $E_c$ (with $f'_c$ in psi)
    - normal weight concrete (~ 145 lb/ft³)
      $$E_c = 57,000 \sqrt{f'_c}$$
    - concrete between 90 and 155 lb/ft³
      $$E_c = 1.5 w_c \sqrt{f'_c}$$
  - cracked
    - I cracked
    - $E$ adjusted

Deflection Limits

- relate to whether or not beam supports or is attached to a damageable non-structural element
- need to check service live load and long term deflection against these

| L/180 | roof systems (typical) – live |
| L/240 | floor systems (typical) – live + long term |
| L/360 | supporting plaster – live |
| L/480 | supporting masonry – live + long term |

Prestressed Concrete

- impose a longitudinal force on a member in order to withstand more loading until the member reaches a tensile limit
Prestressed Concrete

- pretensioned
  - reinforcement bonded
- post-tensioned
  - bonded or unbonded
  - end bearing

- precast
  - concrete premade in a position other than its final position in the structure

Prestressed Concrete

- high strength tendons
  - grade 250
  - grade 270

\[
M(w) = \begin{cases} \frac{P}{A}Mc + \frac{Mc}{I_g}t & \text{if } t > b \\ \frac{P}{A}Mc + \frac{Mc}{I_g}b & \text{if } t \leq b \\ \end{cases}
\]

\[
f'_t = \frac{P}{A} - \frac{Mc}{I_g}
\]

\[
f'_b = -\frac{P}{A} \left(1 + \frac{Mc}{I_g} \right)
\]

\[
\left(\text{remember: } r = \frac{I_g}{A}\right)
\]
Prestressed Concrete

Figure 4.2 Flexural stress distribution throughout loading history. (a) Beam section. (b) Initial prestressing stage. (c) Self-weight and effective prestress. (d) Full dead load plus effective prestress. (e) Full service load plus effective prestress. (f) Limit state of stress at ultimate load for underreinforced beam.

Composite Beams
- concrete  
  - in compression
- steel  
  - in tension
- shear studs

Continuous Beams
- reduced size
- reduced moments
- moments can reverse with loading patterns
- need top & bottom reinforcement
- sensitive to settlement
Approximate Depths

Concrete Columns

- **columns require**
  - ties or spiral reinforcement to “confine” concrete (#3 bars minimum)
  - minimum amount of longitudinal steel (#5 bars minimum: 4 with ties, 5 with spiral)

**Concrete Columns**

- effective length in monolithic casts must be found with respect to stiffness of joint
- not slender when
  \[
  \frac{kL_u}{r} < 22
  \]
Concrete Columns

- $P_o$ – no bending
  
  \[ P_o = 0.85 f'_c (A_g - A_{st}) + f_y A_{st} \]

- $\phi_c = 0.65$ for ties with $P_n = 0.8P_o$
- $\phi_c = 0.70$ for spirals with $P_n = 0.85P_o$

- $P_u \leq \phi_c P_n$

- nominal axial capacity:
  - presumes steel yields
  - concrete at ultimate stress

Columns with Bending

- eccentric loads can cause moments
- moments can change shape and induce more deflection ($P - \Delta$)

- for ultimate strength behavior, ultimate strains can’t be exceeded
  - concrete $0.003$
  - steel $\frac{f_y}{E_s}$

- $P$ reduces with $M$

Columns with Bending

- need to consider combined stresses
  
  \[ \frac{P_n}{P_o} + \frac{M_n}{M_o} \leq 1 \]

- plot interaction diagram
Concrete Floor Systems

- types & spanning direction

Concrete Floor Systems

- flexure design as T-beams (+/- M)
- increase of 10% $V_c$ permitted
- one-way and two-way moments
- slabs need steel
- effective width is
  - $L/4$
  - $b_w + 16t$
  - center-to-center of beams

One-way Joists

- standard stems
- 2.5” to 4.5” slab
- ~30” widths
- reusable forms

**Figure 9.3.1** Actual and equivalent stress distribution over flange width.
Construction Supervision

- proper placement of all reinforcement
  - welding
  - splices

- mix design
  - slump
  - in-situ strength
    - cast cylinders
    - cylinder cores – if needed