Concrete Beam Design

• composite of concrete and steel
• American Concrete Institute (ACI)
  – design for maximum stresses
  – limit state design
    • service loads x load factors
    • concrete holds no tension
    • failure criteria is yield of reinforcement
    • failure capacity x reduction factor
    • factored loads < reduced capacity
  – concrete strength = $f'_{c}$

Concrete Construction

• cast-in-place
• tilt-up
• prestressing
• post-tensioning

Concrete

• low strength to weight ratio
• relatively inexpensive
  – Portland cement
  – aggregate
  – water
• hydration
• fire resistant
• creep & shrink
Reinforcement

• deformed steel bars (rebar)
  – Grade 40, $F_y = 40$ ksi
  – Grade 60, $F_y = 60$ ksi - most common
  – Grade 75, $F_y = 75$ ksi
  – US customary in # of 1/8” $\phi$

• longitudinally placed
  – bottom
  – top for compression reinforcement
  – spliced, hooked, terminated...

Transformation of Material

• $n$ is the ratio of $E$'s
  \[ n = \frac{E_2}{E_1} \]

  • effectively widens a material to get 
    same stress distribution

Behavior of Composite Members

• plane sections remain plane

• stress distribution changes

\[ f_1 = E_1 \varepsilon = -\frac{E_1 y}{\rho} \]

\[ f_2 = E_2 \varepsilon = -\frac{E_2 y}{\rho} \]

Stresses in Composite Section

• with a section transformed to one 
  material, new $I$

  – stresses in that 
    material are 
    determined as usual

  – stresses in the other 
    material need to be 
    adjusted by $n$

\[ n = \frac{E_2}{E_1} = \frac{E_{\text{steel}}}{E_{\text{concrete}}} \]

\[ f_c = -\frac{M_y}{I_{\text{transformed}}} \]

\[ f_s = -\frac{M_y n}{I_{\text{transformed}}} \]
Reinforced Concrete - stress/strain

- Stress distribution
  - Stresses in the concrete above the neutral axis are compressive and non-linearly distributed. In the tension zone below the neutral axis, the concrete is assumed to be linear and the tensile force is taken up by reinforcing steel.

- Typical stress-strain curve for concrete

Reinforced Concrete Analysis

- For stress calculations
  - Steel is transformed to concrete
  - Concrete is in compression above n.a. and represented by an equivalent stress block
  - Concrete takes no tension
  - Steel takes tension
  - Force ductile failure

Location of n.a.

- Ignore concrete below n.a.
- Transform steel
- Same area moments, solve for x

\[ bx \cdot \frac{x}{2} - nA_s (d - x) = 0 \]

T sections

- N.a. equation is different if n.a. below flange

\[ b_h h_f \left( x - \frac{h_f}{2} \right) + (x - h_f) b_w \frac{(x - h_f)}{2} - nA_s (d - x) = 0 \]
ACI Load Combinations*

- 1.4D
- 1.2D + 1.6L + 0.5(L_r or S or R)
- 1.2D + 1.6(L_r or S or R) + (1.0L or 0.5W)
- 1.2D + 1.0W + 1.0L + 0.5(L_r or S or R)
- 1.2D + 1.0E + 1.0L + 0.2S
- 0.9D + 1.0W
- 0.9D + 1.0E

*can also use old ACI factors

Reinforced Concrete Design

- stress distribution in bending

![Diagram of stress distribution in bending]

Wang & Salmon, Chapter 3

Force Equations

- \( C = 0.85 f'_c ba \)
- \( T = A_s f_y \)
- where
  - \( f'_c \) = concrete compressive strength
  - \( a \) = height of stress block
  - \( \beta_1 \) = factor based on \( f'_c \)
  - \( x \) = location to the n.a.
  - \( b \) = width of stress block
  - \( f_y \) = steel yield strength
  - \( A_s \) = area of steel reinforcement

Equilibrium

- \( T = C \)
- \( M_n = T(d-a/2) \)
  - \( d \) = depth to the steel n.a.
- with \( A_s \)
  - \( a = \frac{A_s f_y}{0.85 f'_c b} \)
  - \( M_u \leq \phi M_n \) \( \phi = 0.9 \) for flexure
  - \( M_u = \phi T(d-a/2) = \phi A_s f_y (d-a/2) \)
Over and Under-reinforcement

- over-reinforced
  - steel won’t yield
- under-reinforced
  - steel will yield
- reinforcement ratio
  \[ \rho = \frac{A_s}{bd} \]
  - use as a design estimate to find \( A_s, b, d \)
  - \( \text{max } \rho \) is found with \( \epsilon_{\text{steel}} \geq 0.004 \) (not \( \rho_{\text{bal}} \))

\[ A_s \text{ for a Given Section} \]

- several methods
  - guess a and iterate
    1. guess \( a \) (less than n.a.)
    2. \( A_s = \frac{0.85 f'_c ba}{f_y} \)
    3. solve for \( a \) from \( M_u = \phi A_s f_y (d-a/2) \)
      \[ a = 2 \left( d - \frac{M_u}{\phi A_s f_y} \right) \]
    4. repeat from 2. until \( a \) from 3. matches \( a \) in 2.

\[ A_s \text{ for a Given Section (cont)} \]

- chart method
  - Wang & Salmon Fig. 3.8.1 \( R_n \) vs. \( \rho \)
    1. calculate \( R_n = \frac{M_n}{bd^2} \)
    2. find curve for \( f'_c \) and \( f_y \) to get \( \rho \)
    3. calculate \( A_s \) and \( a \)
- simplify by setting \( h = 1.1d \)

Reinforcement

- min for crack control
- required
  \[ A_s = \frac{3 \sqrt{f'_c}}{f_y} (bd) \]
- not less than
  \[ A_s = \frac{200}{f_y} (bd) \]
- \( A_{s\text{-max}} \): \( a = \beta_1(0.375d) \)
- typical cover
  - 1.5 in, 3 in with soil
- bar spacing
Approximate Depths