Shear in Concrete Beams

- flexure combines with shear to form diagonal cracks
- horizontal reinforcement doesn’t help
- stirrups = vertical reinforcement

ACI Shear Values

- $V_u$ is at distance $d$ from face of support
- shear capacity: $V_c = \nu_c \times b_w d$
  - where $b_w$ means thickness of web at n.a.

- shear stress (beams)
  - $\nu_c = 2\sqrt{f'_c}$
  - $\phi = 0.75$ for shear
  - $f'_c$ is in psi
  - $\phi V_c = \phi 2\sqrt{f'_c} b_w d$

- shear strength:
  - $V_u \leq \phi V_c + \phi V_s$
  - $V_s$ is strength from stirrup reinforcement
**Stirrup Reinforcement**

- **shear capacity:**
  \[ V_s = \frac{A_v f_y d}{s} \]
  - \( A_v \) = area in all legs of stirrups
  - \( s \) = spacing of stirrup

- may need stirrups when concrete has enough strength!

**Required Stirrup Reinforcement**

- **spacing limits**

<table>
<thead>
<tr>
<th>( V_s \leq \frac{f_{cd}}{2} )</th>
<th>( \frac{V_s}{V_u} &lt; \frac{2}{3} )</th>
<th>( \frac{V_u}{V_d} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0 \leq \phi \leq 1 )</td>
<td>( \phi = \frac{V_s}{V_d} )</td>
<td>( \frac{V_u}{V_d} )</td>
</tr>
</tbody>
</table>

- **Recommended minimum**
  - stirrup spacing, \( s \)
  - \( s \geq 0.75 \) ft

- **Maximum**
  - \( s \geq 5 \) in

**Torsional Stress & Strain**

- can see torsional stresses & twisting of axi-symmetrical cross sections
  - torque
  - remain plane
  - undistorted
  - rotates

- not true for square sections....

**Shear Stress Distribution**

- depend on the deformation
  - \( \phi \) = angle of twist
  - measure

- can prove planar section doesn’t distort
**Shearing Strain**

- related to $\phi$ 
  \[ \gamma = \frac{\rho \phi}{L} \]
- $\rho$ is the radial distance from the centroid to the point under strain
- shear strain varies linearly along the radius: $\gamma_{\text{max}}$ is at outer diameter

**Torsional Stress - Strain**

- know $f_v = \tau = G \cdot \gamma$ and $\gamma = \frac{\rho \phi}{L}$
- so $\tau = G \cdot \frac{\rho \phi}{L}$
- where $G$ is the Shear Modulus

**Torsional Stress - Strain**

- from 
  \[ T = \Sigma \tau(\rho) \Delta A \]
- can derive 
  \[ T = \frac{\tau J}{\rho} \]
  
  – where $J$ is the polar moment of inertia
  
  – elastic range 
  \[ \tau = \frac{T \rho}{J} \]

**Shear Stress**

- $\tau_{\text{max}}$ happens at outer diameter

- combined shear and axial stresses
  
  – maximum shear stress at 45° “twisted” plane
Shear Strain

- knowing \( \tau = G \cdot \frac{\rho \phi}{L} \) and \( \tau = \frac{T \rho}{J} \)
- solve: \( \phi = \frac{TL}{JG} \)
- composite shafts: \( \phi = \sum_i \frac{T_i L_i}{J_i G_i} \)

Noncircular Shapes

- torsion depends on \( J \)
- plane sections don’t remain plane
- \( \tau_{\text{max}} \) is still at outer diameter

\[
\tau_{\text{max}} = \frac{T}{c_1 ab^2} \quad \phi = \frac{TL}{c_2 ab^3 G}
\]

- where \( a \) is longer side (> \( b \))

Open Thin-Walled Sections

- with very large \( a/b \) ratios:

\[
\tau_{\text{max}} = \frac{T}{\frac{1}{3} ab^2} \quad \phi = \frac{TL}{\frac{1}{3} ab^3 G}
\]

Shear Flow in Closed Sections

- \( q \) is the internal shear force/unit length

\[
\tau = \frac{T}{2 t a} \quad \phi = \frac{TL}{4 t a^2} \sum_i \frac{s_i}{t_i}
\]

- \( a \) is the area bounded by the centerline
- \( s_i \) is the length segment, \( t_i \) is the thickness
Shear Flow in Open Sections
• each segment has proportion of $T$ with respect to torsional rigidity,
\[
\tau_{\text{max}} = \frac{Tt_{\text{max}}}{\frac{1}{3} \sum b_i t_i^3}
\]

• total angle of twist:
\[
\phi = \frac{TL}{\frac{1}{3} G \sum b_i t_i^3}
\]

• I beams - web is thicker, so $\tau_{\text{max}}$ is in web

Torsional Shear Stress
• twisting moment
• and beam shear

Torsional Shear Reinforcement
• closed stirrups
• more longitudinal reinforcement
• area enclosed by shear flow

Development Lengths
• required to allow steel to yield ($f_y$)
• standard hooks
  – moment at beam end
• splices
  – lapped
  – mechanical connectors
Development Lengths
- \( l_d \), embedment required both sides
- proper cover, spacing:
  - No. 6 or smaller
    \[ l_d = \frac{d_b F_y}{25 \sqrt{f'_c}} \text{ or 12 in. minimum} \]
  - No. 7 or larger
    \[ l_d = \frac{d_b F_y}{20 \sqrt{f'_c}} \text{ or 12 in. minimum} \]

Concrete Deflections
- elastic range
  - I transformed
  - \( E_c \) (with \( f'_c \) in psi)
    - normal weight concrete (~ 145 lb/ft\(^3\))
      \[ E_c = 57,000 \sqrt{f'_c} \]
    - concrete between 90 and 160 lb/ft\(^3\)
      \[ E_c = w_c^{1.5} 33 \sqrt{f'_c} \]
- cracked
  - I cracked
  - \( E \) adjusted

Development Lengths
- hooks
  - bend and extension
    \[ l_{dh} = \frac{1200 d_b}{\sqrt{f'_c}} \]

Development Lengths
- bars in compression
  \[ l_d = \frac{0.02 d_b F_y}{\sqrt{f'_c}} \leq 0.0003 d_b F_y \]
- splices
  - tension minimum is function of \( l_d \) and splice classification
  - compression minimum
  - is function of \( d_b \) and \( F_y \)
Deflection Limits

• relate to whether or not beam supports or is attached to a damageable non-structural element

• need to check service live load and long term deflection against these

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<tbody>
<tr>
<td>L/180</td>
<td>roof systems (typical) – live</td>
</tr>
<tr>
<td>L/240</td>
<td>floor systems (typical) – live + long term</td>
</tr>
<tr>
<td>L/360</td>
<td>supporting plaster – live</td>
</tr>
<tr>
<td>L/480</td>
<td>supporting masonry – live + long term</td>
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