**Elements of Architectural Structures:**

**Form, Behavior, and Design**

ARCH 614

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**Spring 2013**

**Lecture eight**

**Beam sections - geometric properties**

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**Center of Gravity**

- **location of equivalent weight**
- **determined with calculus**

\[ W = \int dW \]

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**Centroid**

- **“average” x & y of an area**
- **for a volume of constant thickness**
  
  \[ \Delta W = \gamma t \Delta A \quad \text{where } \gamma \text{ is weight/volume} \]

  
  \[ \text{center of gravity = centroid of area} \]

\[ \bar{x} = \frac{\sum(x \Delta A)}{A} \]

\[ \bar{y} = \frac{\sum(y \Delta A)}{A} \]
**Centroid**

- for a line, sum up length

\[
\bar{x} = \frac{\sum (x\Delta L)}{L}
\]

\[
\bar{y} = \frac{\sum (y\Delta L)}{L}
\]

**1st Moment Area**

- math concept
- the moment of an area about an axis

\[
Q_x = \bar{y}A
\]

\[
Q_y = \bar{x}A
\]

**Symmetric Areas**

- symmetric about an axis
- symmetric about a center point
- mirrored symmetry

**Composite Areas**

- made up of basic shapes
- areas can be negative
- (centroids can be negative for any area)
Basic Procedure
1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table
5. Fill in table
6. Sum necessary columns
7. Calculate $\hat{x}$ and $\hat{y}$

<table>
<thead>
<tr>
<th>Component</th>
<th>Area</th>
<th>$\bar{x}$</th>
<th>$\bar{y}$</th>
<th>$\bar{y}A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Area Centroids
• Figure A.1 – pg 598

Moments of Inertia
• 2nd moment area
  – math concept
  – area $x$ (distance)$^2$
• need for behavior of
  – beams
  – columns

Moment of Inertia
• about any reference axis
• can be negative

$$I_y = \sum x_i^2 \Delta A = \int x^2 dA$$

$$I_x = \sum y_i^2 \Delta A = \int y^2 dA$$

(or $I_{x-x} = \sum z^2 a$)
• resistance to bending and buckling
**Moment of Inertia**

- same area moved away a distance
  - larger $I$

![Image of Moment of Inertia](image1)

**Polar Moment of Inertia**

- for roundish shapes
- uses polar coordinates ($r$ and $\theta$)
- resistance to twisting

$$J_o = \int r^2 dA$$

![Image of Polar Moment of Inertia](image2)

**Radius of Gyration**

- measure of inertia with respect to area

$$r_x = \sqrt{\frac{I_x}{A}}$$

![Image of Radius of Gyration](image3)

**Parallel Axis Theorem**

- can find composite $I$ once composite centroid is known (basic shapes)

$$I = I_o + Az^2 = \bar{I}_x + Ad_y^2$$

$$I = \sum \bar{I} + \sum Ad^2$$

$$\bar{I} = I - Ad^2$$

![Image of Parallel Axis Theorem](image4)
Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table with A, $\bar{x}$, $\bar{y}$A, $\bar{I}$’s, d’s, and $Ad^2$’s
5. Fill in table and get $\hat{x}$ and $\hat{y}$ for composite
6. Sum necessary columns
7. Sum I’s and Ad2’s

\[ d_x = \hat{x} - \bar{x} \]
\[ d_y = \hat{y} - \bar{y} \]

Area Moments of Inertia

- Figure A.11 – pg. 611: (bars refer to centroid)
  - x, y
  - x’, y’
  - C