ARCH 614: Practice Quiz 8

Note: No aids are allowed for part 1. One side of a letter sized paper with notes is allowed during part 2, along with a silent, **non-programmable** calculator. There are reference charts on pages 2-6 for part 2.

Clearly show your work and answer.

**Part 1) Worth 5 points** (conceptual questions)

**Part 2) Worth 45 points**

*(NOTE: The loading type [ex, live, dead, wind...] and sizes can and will be changed for the quiz with respect to the beam diagrams and formula provided.)*

A wide flange beam of A992 steel ($F_y = 50$ ksi, $E = 30 \times 10^3$ ksi) is needed to span 32 ft and support uniformly distributed loads of $850$ lb/ft of dead load (from materials), the self weight, and $1150$ lb/ft of linearly distributed live load. The beam is simply supported with a maximum unbraced length of 11 ft.

a) Select the most economical beam based on flexural strength using the provided chart (including self weight). *Assume that the dead load will determine the location of the maximum bending moment and superimpose the live load moment at that location.*

b) If a W21 x 44 ($A = 13.0$ in.$^2$, $d = 20.66$ in., $t_w = 0.35$ in., $b_f = 6.50$ in., $t_f = 0.45$ in., $I_x = 843$ in.$^4$) is chosen, is it adequate for shear with a self weight of 44 lb/ft?

c) Determine the moment of inertia required such that the total [or live load or dead load] deflection, ignoring self weight, does not exceed 1.25 inches.

*Answers – Not provided on actual quiz!*

a) $M_u = 248.3$ k-ft, use W14x48 ($M_{u*} > 250.5$ k-ft)
b) $V_u = 56.8$ k, $\phi V_u = 216.9$ k, \( \therefore \) OK

c) $I_{req'd} = 897$ in.$^4$ [$I_{req'd-dead} = 535$ in.$^4$, $I_{req'd-live} = 362.3$ in.$^4$]
1. **SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD**

   Total Equiv. Uniform Load \( = \frac{w}{l} \)
   
   \[ R = V = \frac{w}{2} \]
   
   \[ V_x = w \left( \frac{l}{2} - x \right) \]
   
   \[ M_{max. \atop \text{at center}} = \frac{wx}{8} (l-x) \]
   
   \[ \Delta x_{max. \atop \text{at center}} = -\frac{wx}{384EI} \left( l^2 - 2lx^2 + x^3 \right) \]

2. **SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO ONE END**

   Total Equiv. Uniform Load \( = 16W = 1.0264W \)
   
   \[ R_1 = V_1 = \frac{8W}{3} \]
   
   \[ R_2 = V_2 = \frac{8W}{3} \]
   
   \[ V_x = \frac{Wx^2}{3} \]
   
   \[ M_{max. \atop \text{at } x = \frac{l}{2} = 0.5774l} = \frac{Wx^2}{2} \]
   
   \[ M_x = \frac{Wx}{3} (l-x) \]
   
   \[ M_{max. \atop \text{at center}} = \frac{Wx}{384EI} \left( l^2 - 2lx^2 + x^3 \right) \]
   
   \[ \Delta x_{max. \atop \text{at center}} = \frac{Wx}{768EI} \left( 3l^2 - 10lx^2 + 7l^3 \right) \]

3. **SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO CENTER**

   Total Equiv. Uniform Load \( = 4W = 2W \)
   
   \[ R = V = \frac{W}{2} \]
   
   \[ V_x = \frac{W}{2} \left( l - 4x^2 \right) \]
   
   \[ M_{max. \atop \text{at center}} = \frac{Wl}{6} \]
   
   \[ M_x = \frac{Wx}{2} \left( 1 - 2x^2 - \frac{3x^3}{2} \right) \]
   
   \[ \Delta x_{max. \atop \text{at center}} = \frac{Wx}{480EI \sqrt{3}} \left( 5l^3 - 6l^2 x^2 \right) \]

4. **SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED**

   \[ R_1 = V_1 \left( \text{max. when } a < c \right) = \frac{wa}{2l} \]
   
   \[ R_2 = V_2 \left( \text{max. when } a > c \right) = \frac{wb}{2l} \]
   
   \[ M_{max. \atop \text{at } x = a + \frac{R_1}{2w}} = \frac{R_1 (a + R_1)}{20l} \]
   
   \[ M_x = \frac{R_1 x^{\frac{1}{2}} (l-x)}{2} \]

5. **SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT ONE END**

   \[ R_1 = V_1 \left( \text{max.} \right) = \frac{wa}{2l} \]
   
   \[ R_2 = V_2 = \frac{wa}{2l} \]
   
   \[ V_x = \frac{wx}{2} \]
   
   \[ M_{max. \atop \text{at } x = \frac{R_1}{w}} = \frac{R_1 x^2}{2w} \]
   
   \[ M_x = \frac{R_1 x^{\frac{1}{2}} (l-x)}{2} \]

6. **SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT EACH END**

   \[ R_1 = V_1 = \frac{wa\sqrt{2} \left( a - l \right) + 2l \sqrt{3} x}{2l} \]
   
   \[ R_2 = V_2 = \frac{wa\sqrt{2} \left( l - a \right) + 2l \sqrt{3} x}{2l} \]
   
   \[ V_x = \frac{wx}{2} \]
   
   \[ M_{max. \atop \text{at } x = \frac{R_1}{w}} = \frac{R_1 x}{2l} \]
   
   \[ M_x = \frac{R_1 x \sqrt{3} (l-x)}{2} \]

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**REFERENCE CHARTS FOR QUIZ 8**
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7. **SIMPLE BEAM—CONCENTRATED LOAD AT CENTER**

\[ \text{Total Equiv. Uniform Load} = 2P \]
\[ R = V = \frac{P}{2} \]
\[ M_{\text{max}} \text{ (at point of load)} = \frac{P}{4} \]
\[ M_x \text{ (when } x < \frac{l}{2} \text{)} = \frac{Px}{2} \]
\[ \Delta_{\text{max}} \text{ (at point of load)} = \frac{P^2 l}{48EI} \]
\[ \Delta_x \text{ (when } x < \frac{l}{2} \text{)} = \frac{Px}{48EI}(3x^2 - 4x) \]

8. **SIMPLE BEAM—CONCENTRATED LOAD AT ANY POINT**

\[ \text{Total Equiv. Uniform Load} = \frac{8}{l} \frac{Pab}{a} \]
\[ R_1 = V_1 \text{ (max. when } a < b \text{)} = \frac{Pa}{l} \]
\[ R_2 = V_2 \text{ (max. when } a > b \text{)} = \frac{Pb}{l} \]
\[ M_{\text{max}} \text{ (at point of load)} = \frac{Pbx}{l} \]
\[ \Delta_{\text{max}} \text{ (at } x = \frac{a}{2} \text{ when } a > b \text{)} = \frac{Pab(a + 2b)}{27EI l} \]
\[ \Delta_a \text{ (at point of load)} = \frac{Pabx}{27EI l} \]
\[ \Delta_x \text{ (when } x < a \text{)} = \frac{Pbx(3a - b^2 - x^2)}{6EI l} \]

9. **SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED**

\[ \text{Total Equiv. Uniform Load} = \frac{8}{l} \frac{P}{a} \]
\[ R = V = P \]
\[ M_{\text{max}} \text{ (between loads)} = Pa \]
\[ M_x \text{ (when } x < a \text{)} = \frac{Px}{24EI}(3x^2 - 4a^2) \]
\[ \Delta_{\text{max}} \text{ (at center)} = \frac{Pa}{24EI}(3a^2 - 4a^2) \]
\[ \Delta_x \text{ (when } x < a \text{)} = \frac{Px}{6EI}(3a^2 - 3a^2 - a^2) \]
\[ \Delta_x \text{ (when } x > a \text{ and } (l - a) \text{)} = \frac{Pa}{6EI}(3a^2 - 3x^2 - a^2) \]

10. **SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED**

\[ R_1 = V_1 \text{ (max. when } a < b \text{)} = \frac{P}{l}(l - a + b) \]
\[ R_2 = V_2 \text{ (max. when } a > b \text{)} = \frac{P}{l}(l - b + a) \]
\[ V_x \text{ (when } x > a \text{ and } < (l - b) \text{)} = \frac{P}{l}(b - a) \]
\[ M_1 \text{ (max. when } a < b \text{)} = R_1 a \]
\[ M_2 \text{ (max. when } a > b \text{)} = R_2 b \]
\[ M_x \text{ (when } x < a \text{)} = R_1 x \]
\[ M_x \text{ (when } x > a \text{ and } < (l - b) \text{)} = R_1 x - P(x - a) \]

11. **SIMPLE BEAM—TWO UNEQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED**

\[ R_1 = V_1 \]
\[ R_2 = V_2 \text{ (when } x > a \text{ and } < (l - b) \text{)} = R_1 - P_1 \]
\[ M_1 \text{ (max. when } R_1 < P_1 \text{)} = R_1 a \]
\[ M_2 \text{ (max. when } R_2 < P_2 \text{)} = R_2 b \]
\[ M_x \text{ (when } x < a \text{)} = R_1 x \]
\[ M_x \text{ (when } x > a \text{ and } < (l - b) \text{)} = R_1 x - P_1(x - a) \]

12. **BEAM FIXED AT ONE END, SUPPORTED AT OTHER—UNIFORMLY DISTRIBUTED LOAD**

\[ \text{Total Equiv. Uniform Load} = \frac{u}{l} \]
\[ R_1 = V_1 = \frac{2u}{l} \]
\[ R_2 = V_2 = \frac{5u}{l} \]
\[ V_x = \frac{u}{l} \]
\[ M_1 \text{ (at } x = \frac{9}{8} \text{)} = \frac{u^2}{8} \]
\[ M_2 \text{ (at } x = \frac{15}{8} \text{)} = \frac{u^2}{8} \]
\[ M_{\text{max}} \text{ (at } x = \frac{1}{16} (1 + \sqrt{33}) = 4215 \text{)} \]
\[ \Delta_{\text{max}} \text{ (at } x = \frac{1}{16} (1 + \sqrt{33}) = 4215 \text{)} \]
13. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—
CONCENTRATED LOAD AT CENTER

Total Equiv. Uniform Load = \( \frac{3P}{2} \)

\( R_1 = V_1 = \frac{5P}{16} \)

\( R_2 = V_2 \text{ max} = \frac{11P}{16} \)

\( M_{\text{max}} \text{ at fixed end} = \frac{3P}{16} \)

\( M_1 \text{ at point of load} = \frac{32P}{16} \)

\( M_x \text{ when } x < \frac{1}{2} = \frac{5Px}{16} \)

\( M_x \text{ when } x > \frac{1}{2} = P \left( \frac{1}{2} - \frac{11x}{16} \right) \)

\( \Delta_{\text{max}} \text{ at } x = \frac{1}{5} = 4.4721 \)

\( \frac{P^2}{16} \left( \frac{1}{5} - 0.00931 \right) \frac{P}{16} \)

\( \Delta_x \text{ at point of load} = \frac{7P}{6} \)

\( \Delta_x \text{ when } x < \frac{1}{2} = \frac{P}{6} (3x^2 - 5x^2) \)

\( \Delta_x \text{ when } x > \frac{1}{2} = \frac{P}{96EI} (x-1)^2 (11x-2) \)

14. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—
CONCENTRATED LOAD AT ANY POINT

\( R_1 = V_1 = 2P \left( a + 2l \right) \)

\( R_2 = V_2 = \frac{Pa}{2} \left( 3a - a^2 \right) \)

\( M_1 \text{ at point of load} = ax \)

\( M_2 \text{ at fixed end} = \frac{Pa}{2} (a + l) \)

\( M_x \text{ when } x < a = R_1x \)

\( M_x \text{ when } x > a = R_1x - P(x - a) \)

\( \Delta_{\text{max}} \text{ when } x < \frac{1}{2} = \frac{P}{3EI} \left( \frac{1}{2} - a^2 \right) \)

\( \Delta_{\text{max}} \text{ when } x > \frac{1}{2} = \frac{P}{6EI} \sqrt{\frac{a}{2l}} \)

\( \Delta_s \text{ at point of load} = \frac{Pab^3}{12EI (3l + a)} \)

\( \Delta_x \text{ when } x < a = \frac{Pa}{12EI (3l + a)} \left( 12x^2 - 2l^2 - ax^2 \right) \)

\( \Delta_x \text{ when } x > a = \frac{Pa}{12EI (3l + a)} \left( l-x \right)^2 (3l-x-a^2 - 2a^2) \)

15. BEAM FIXED AT BOTH ENDS—UNIFORMLY DISTRIBUTED LOADS

Total Equiv. Uniform Load = \( \frac{2wL}{3} \)

\( R = V = \frac{w}{2} \)

\( V_x = \frac{w}{2} \left( \frac{1}{2} - x \right) \)

\( M_{\text{max}} \text{ at ends} = \frac{wL}{12} \)

\( M_1 \text{ at center} = \frac{wL}{24} \)

\( M_x = \frac{w}{6} (L - x - 2x^2) \)

\( \Delta_{\text{max}} \text{ at center} = \frac{384EI}{wL} \)

\( \Delta_x = \frac{wL^2}{24EI} \left( 1 - x \right)^2 \)

16. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT CENTER

Total Equiv. Uniform Load = \( P \)

\( R = V = \frac{P}{2} \)

\( V_x = \frac{P}{2} \)

\( M_{\text{max}} \text{ at center and ends} = \frac{P}{3} \)

\( M_x \text{ when } x < \frac{1}{2} = \frac{P}{192EI} \left( 4x - 1 \right) \)

\( M_x \text{ when } x > \frac{1}{2} = \frac{P}{48EI} (3l - 4x) \)

\( \Delta_{\text{max}} \text{ at center} = \frac{192EI}{P} \)

\( \Delta_x \text{ when } x < \frac{1}{2} = \frac{P}{48EI} (3l - 4x) \)

\( \Delta_x \text{ when } x > \frac{1}{2} = \frac{P}{48EI} (3l - 4x) \)

17. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT ANY POINT

\( R_1 = V_1 \text{ max, when } a < b = \frac{Pb}{l} - \left( 3a + b \right) \)

\( R_2 = V_2 \text{ max, when } a > b = \frac{Pa}{l} - \left( a + 3b \right) \)

\( M_1 \text{ max, when } a < b = \frac{Pab}{l} \)

\( M_2 \text{ max, when } a > b = \frac{Pab}{l} \)

\( M_x \text{ at point of load} = \frac{2Pab}{l} \)

\( M_x \text{ when } x < a = \frac{Rx - Pab}{l} \)

\( M_x \text{ when } x > a = \frac{Pab}{l} \left( 3a - b \right) \)

\( \Delta_{\text{max}} \text{ when } a > b \text{ at } x = \frac{3a + b}{3l} \)

\( \Delta_{\text{max}} \text{ when } a < b \text{ at } x = \frac{3a - b}{3l} \)

\( \Delta_{\text{max}} \text{ at point of load} = \frac{3l^2 (3a + b)^2}{Pab^2} \)

\( \Delta_x \text{ when } x < a = \frac{Pb}{6EI} \left( 3a - 3ax - bx \right) \)

\( \Delta_x \text{ when } x > a = \frac{Pb}{6EI} \left( 3l - 3ax - bx \right) \)
REFERENCE CHARTS FOR QUIZ 8

18. CANTILEVER BEAM—LOAD INCREASING UNIFORMLY TO FIXED END

- Total Equiv. Uniform Load: $\frac{8}{3} W$
- $R = V = \frac{W}{3}$
- $V_x = \frac{Wx^2}{2}$
- $M_{\text{max.}} (\text{at fixed end}) = \frac{Wl}{3}$
- $M_{x} (\text{at free end}) = \frac{Wx^2}{3l}$
- $\Delta_{\text{max.}} (\text{at free end}) = \frac{Wl^3}{18EI}$
- $\Delta_{x} = \frac{Wl}{36EI/l^2} (x^2 - 5lx + 4l^2)$

19. CANTILEVER BEAM—UNIFORMLY DISTRIBUTED LOAD

- Total Equiv. Uniform Load: $4wl$
- $R = V = \frac{w}{3}$
- $V_x = \frac{wx}{2}$
- $M_{\text{max.}} (\text{at fixed end}) = \frac{wx^2}{2}$
- $M_{x} (\text{at free end}) = \frac{wx^2}{8EI}$
- $\Delta_{\text{max.}} (\text{at free end}) = \frac{wx^4}{24EI} (x^4 - 4lx^3 + 3l^4)$
- $\Delta_{x} = \frac{wx^4}{24EI} (x^4 - 4lx^3 + 3l^4)$

20. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—UNIFORMLY DISTRIBUTED LOAD

- Total Equiv. Uniform Load: $\frac{8}{3} w$l
- $R = V = \frac{w}{3}$
- $V_x = \frac{wx}{6}$
- $M_{\text{max.}} (\text{at fixed end}) = \frac{wx^2}{6}$
- $M_{x} (\text{at deflected end}) = \frac{wx^2}{6} (2l - 3lx)$
- $M_{max.} (\text{at deflected end}) = \frac{wx^4}{24EI}$
- $\Delta_{max.} (\text{at deflected end}) = \frac{wx^4}{24EI}$
- $\Delta_{x} = \frac{wx^4}{24EI} (l^2 - x^2)^2$

21. CANTILEVER BEAM—CONCENTRATED LOAD AT ANY POINT

- Total Equiv. Uniform Load: $\frac{8Pb}{l}$
- $R = V = P$
- $M_{\text{max.}} (\text{at fixed end}) = Pb$
- $M_{x} (\text{when } x > a) = P (x - a)$
- $\Delta_{\text{max.}} (\text{at free end}) = \frac{Pb^2}{EI}$
- $\Delta_{x} (\text{at point of load}) = \frac{Pb^2}{3EI}$
- $\Delta_{x} (\text{when } x < a) = \frac{Pb^2}{6EI} (3l - 3x - b)$
- $\Delta_{x} (\text{when } x > a) = \frac{Pb^2}{6EI} (l + x)$

22. CANTILEVER BEAM—CONCENTRATED LOAD AT FREE END

- Total Equiv. Uniform Load: $8P$
- $R = V = P$
- $M_{\text{max.}} (\text{at fixed end}) = Pl$
- $M_{x} = Px$
- $\Delta_{\text{max.}} (\text{at free end}) = \frac{P^3}{3EI}$
- $\Delta_{x} (\text{when } x < a) = \frac{P}{6EI} (2l - 3lx^3 + x^4)$

23. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—CONCENTRATED LOAD AT DEFLECTED END

- Total Equiv. Uniform Load: $4P$
- $R = V = P$
- $M_{\text{max.}} (\text{at both ends}) = \frac{Pl}{2}$
- $M_{x} = \frac{P}{2} (1 - x)$
- $\Delta_{\text{max.}} (\text{at deflected end}) = \frac{P^3}{12EI}$
- $\Delta_{x} = \frac{P}{12EI} (l - x)^2 (l + 2x)$