moment of inertia of an area
Moments of Inertia

• 2nd moment area
  – math concept
  – area x (distance)^2

• need for behavior of
  – beams
  – columns
Moment of Inertia

- about any reference axis
- can be negative

\[ I_y = \int x^2 \, dA \]
\[ I_x = \int y^2 \, dA \]

- resistance to bending and buckling
Moment of Inertia

• same area moved away a distance – larger $I$
Polar Moment of Inertia

• for round-ish shapes
• uses polar coordinates \((r \text{ and } \theta)\)
• resistance to twisting

\[
J_o = \int r^2 \, dA
\]
Radius of Gyration

• measure of inertia with respect to area

\[ r_x = \sqrt{\frac{I_x}{A}} \]
Parallel Axis Theorem

• can find composite \( I \) once composite centroid is known (basic shapes)

\[
I_x = I_{cx} + Ad_y^2
= \bar{I}_x + Ad_y^2
\]

\[
I = \sum \bar{I} + \sum Ad^2
\]

\[
\bar{I} = I - Ad^2
\]
Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table with $A$, $\bar{x}$, $\bar{x}A$, $\bar{y}$, $\bar{y}A$, $\bar{I}$’s, d’s, and $Ad^2$’s
5. Fill in table and get $\hat{x}$ and $\hat{y}$ for composite
6. Sum necessary columns
7. Sum $\bar{I}$’s and $Ad^2$’s

\[ (d_x = \hat{x} - \bar{x}) \]
\[ (d_y = \hat{y} - \bar{y}) \]
Area Moments of Inertia

- Table 7.2 – pg. 252: (bars refer to centroid)
  - $x$, $y$
  - $x'$, $y'$
  - C

<table>
<thead>
<tr>
<th>Shape</th>
<th>$I_x$</th>
<th>$I_y$</th>
<th>$J_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>$\frac{1}{12}bh^3$</td>
<td>$\frac{1}{12}b'h^3$</td>
<td>$\frac{1}{2}bh(b^2 + h^2)$</td>
</tr>
<tr>
<td>Triangle</td>
<td>$\frac{1}{2}bh^3$</td>
<td>$\frac{1}{2}b'h^3$</td>
<td></td>
</tr>
<tr>
<td>Circle</td>
<td>$\frac{1}{4}\pi r^4$</td>
<td>$\frac{1}{2}\pi r^4$</td>
<td></td>
</tr>
</tbody>
</table>