lecture
thirteen

beam forces – internal
Beams

• span horizontally
  – floors
  – bridges
  – roofs

• loaded transversely by gravity loads

• may have internal axial force

• will have internal shear force

• will have internal moment (bending)
Internal Forces

- **trusses**
  - axial only, (compression & tension)

- **in general**
  - axial force
  - shear force, $V$
  - bending moment, $M$
Beam Loading

- concentrated force
- concentrated moment
  - spandrel beams

(d) Pure moment.
Beam Loading

- uniformly distributed load (line load)
- non-uniformly distributed load
  - hydrostatic pressure
  - wind loads
Beam Supports

- **statically determinate**

  - simply supported (most common)
  - overhang
  - cantilever

- **statically indeterminate**

  - continuous (most common case when $L_1=L_2$)
  - Propped
  - Restrained
**Beam Supports**

- *in the real world, modeled type*

(a) Beam supported by a neoprene pad.

(b) Timber beam–column connection with T-plate.
Internal Forces in Beams

- **like method of sections / joints**
  - no axial forces
- **section must be in equilibrium**
- **want to know where biggest internal forces and moments are for designing**
V & M Diagrams

- tool to locate $V_{\text{max}}$ and $M_{\text{max}}$
- necessary for designing
- have a different sign convention than external forces, moments, and reactions
Sign Convention

- shear force, $V$:
  - cut section to LEFT
  - if $\sum F_y$ is positive by statics, $V$ acts down and is POSITIVE
  - beam has to resist shearing apart by $V$
Shear Sign Convention

(+) Shear.

(+) Shear.

(−) Shear.

(−) Shear.
Sign Convention

- bending moment, $M$: 
  - cut section to LEFT
  - if $\sum M_{cut}$ is clockwise, $M$ acts ccw and is POSITIVE – flexes into a “smiley” beam has to resist bending apart by $M$
Bending Moment Sign Convention

(+) Moment.

(−) Moment.

Holds Water

Sheds Water

(+)(−) Moment.
Deflected Shape

- **positive bending moment**
  - tension in bottom, compression in top
- **negative bending moment**
  - tension in top, compression in bottom
- **zero bending moment**
  - inflection point
Constructing V & M Diagrams

- along the beam length, plot V, plot M
Mathematical Method

- cut sections with $x$ as width
- write functions of $V(x)$ and $M(x)$
Method 1: Equilibrium

- cut sections at important places
- plot V & M

\[ V(L) = \begin{cases} + & \text{at } L/2 \\ - & \text{at other sections} \end{cases} \]

\[ M(L) = \begin{cases} + & \text{at } L/2 \\ - & \text{at other sections} \end{cases} \]
Method 1: Equilibrium

- **important places**
  - supports
  - concentrated loads
  - start and end of distributed loads
  - concentrated moments

- **free ends**
  - zero forces
Method 2: Semigraphical

- by knowing
  - area under loading curve = change in $V$
  - area under shear curve = change in $M$
  - concentrated forces cause “jump” in $V$
  - concentrated moments cause “jump” in $M$

\[
V_D - V_C = - \int_{x_C}^{x_D} w \, dx \\
M_D - M_C = \int_{x_C}^{x_D} V \, dx
\]
Method 2

- relationships

Figure 7.11 Relationship of load, shear, moment, slope, and deflection diagrams.
Method 2: Semigraphical

- $M_{\text{max}}$ occurs where $V = 0$ (calculus)

![Diagram showing the concept of maximum bending moment ($M_{\text{max}}$) occurring at $V = 0$ (calculus).](image)
Curve Relationships

- integration of functions
- line with 0 slope, integrates to sloped

- ex: load to shear, shear to moment
Curve Relationships

- *line with slope, integrates to parabola*

- *ex: load to shear, shear to moment*
Curve Relationships

- parabola, integrates to 3\textsuperscript{rd} order curve

- \textit{ex: load to shear, shear to moment}
Basic Procedure

1. Find reaction forces & moments
   Plot axes, underneath beam load diagram

V:

2. Starting at left
3. Shear is 0 at free ends
4. Shear jumps with concentrated load
5. Shear changes with area under load
Basic Procedure

M:
6. Starting at left
7. Moment is 0 at free ends
8. Moment jumps with moment
9. Moment changes with area under V
Triangle Geometry

- slope of $V$ is $w$ (-$w$:1)

\[ x \cdot w = V_A \Rightarrow x = \frac{V_A}{w} \]
Parabolic Shapes

- cases

- up fast, then slow
- up slow, then fast
- down fast, then slow
- down slow, then fast