Architectural Structures I: Statics and Strength of Materials
ENDS 231
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lecture sixteen

elasticity & strain
Deformations

- materials deform
- axially loaded materials change length
- normal stress is load per unit area
- STRAIN:
  - change in length over length
  - UNITLESS

\[ \varepsilon = \frac{\delta}{L} \]
Shearing Strain

- deformations with shear
- parallelogram
- change in angles
- stress: $\tau$
- strain: $\gamma$
  - unitless (radians)

$$\gamma = \frac{\delta_s}{L} = \tan \phi \approx \phi$$
Shearing Strain

- deformations with torsion
- twist
- change in angle of line

stress: \( \tau \)

strain: \( \gamma = \frac{\rho \phi}{L} \)

- unitless (radians)
Load and Deformation
• for stress, need $P$ & $A$
• for strain, need $\delta$ & $L$
  – how?
  – TEST with load and measure
  – plot $P/A$ vs. $\varepsilon$
Material Behavior

- every material has its own response
  - 10,000 psi
  - $L = 10 \text{ in}$
  - Douglas Fir vs. steel?

Figure 5.20  Stress-strain diagram for various materials.
Behavior Types

- ductile - “necking”
- true stress
  \[ f = \frac{P}{A} \]
- engineering stress
  \[ f = \frac{P}{A_o} \]
Behavior Types

• brittle

• semi-brittle
Stress to Strain

• important to us in $f-\varepsilon$ diagrams:
  – straight section
  – LINEAR-ELASTIC
  – recovers shape (no permanent deformation)

Figure 5.20  Stress-strain diagram for various materials.
Hooke’s Law

- straight line has constant slope
- Hooke’s Law

\[ f = E \cdot \varepsilon \]

- \( E \)
  - Modulus of elasticity
  - Young’s modulus
  - units just like stress
Stiffness

- ability to resist strain

- steels
  - same E
  - different yield points
  - different ultimate strength

Figure 5.20  Stress-strain diagram for various materials.
Isotropy & Anisotropy

• **ISOTROPIC**
  – materials with \( E \) **same** at any direction of loading
  – ex. steel

• **ANISOTROPIC**
  – materials with **different** \( E \) at any direction of loading
  – ex. wood is **orthotropic**
Elastic, Plastic, Fatigue

- elastic springs back
- plastic has permanent deformation
- fatigue caused by reversed loading cycles
Plastic Behavior

- ductile

Figure 5.22 Stress-strain diagram for mild steel (A36) with key points highlighted.
Lateral Strain

- or "what happens to the cross section with axial stress"

\[ \varepsilon_x = \frac{f_x}{E} \]

\[ f_y = f_z = 0 \]

- strain in lateral direction
  - negative
  - equal for isometric materials

\[ \varepsilon_y = \varepsilon_z \]
Poisson’s Ratio

- constant relationship between longitudinal strain and lateral strain

\[
\mu = -\frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}
\]

\[
\varepsilon_y = \varepsilon_z = -\frac{\mu f_x}{E}
\]

- sign! \( 0 < \mu < 0.5 \)
Calculating Strain

- **from Hooke’s law**
  \[ f = E \cdot \epsilon \]

- **substitute**
  \[ \frac{P}{A} = E \cdot \frac{\delta}{L} \]

- **get** \( \Rightarrow \)
  \[ \delta = \frac{PL}{AE} \]
Orthotropic Materials

- non-isometric
- directional values of $E$ and $\mu$

- ex:
  - plywood
  - laminates
  - polymer composites
Stress Concentrations

- why we use $f_{ave}$
- increase in stress at changes in geometry
  - sharp notches
  - holes
  - corners

Figure 5.35 Stress trajectories around a hole.
Maximum Stresses

- if we need to know where $\text{max } f$ and $f_v$ happen:

\[ \theta = 0^\circ \rightarrow \cos \theta = 1 \]
\[ f_{\text{max}} = \frac{P}{A_o} \]

\[ \theta = 45^\circ \rightarrow \cos \theta = \sin \theta = \sqrt{0.5} \]
\[ f_{v\text{-max}} = \frac{P}{2A_o} = \frac{f_{\text{max}}}{2} \]
Maximum Stresses

**Fig. 2-37** Shear failure along a 45° plane of a wood block loaded in compression

**Fig. 2-38** Slip bands (or Lüders’ bands) in a polished steel specimen loaded in tension
Design of Members

• beyond allowable stress...
• materials aren’t uniform 100% of the time
  – ultimate strength or capacity to failure may be different and some strengths hard to test for

• RISK & UNCERTAINTY

\[ f_u = \frac{P_u}{A} \]
Factor of Safety

- accommodate uncertainty with a safety factor:

\[
\text{allowable load} = \frac{\text{ultimate load}}{F.S}
\]

- with linear relation between load and stress:

\[
F.S = \frac{\text{ultimate load}}{\text{allowable load}} = \frac{\text{ultimate stress}}{\text{allowable stress}}
\]
Load and Resistance Factor Design

- loads on structures are
  - not constant
  - can be more influential on failure
  - happen more or less often
  - UNCERTAINTY

\[ R_u = \gamma_D R_D + \gamma_L R_L \leq \phi R_n \]

- \( \phi \) - resistance factor
- \( \gamma \) - load factor for (D)ead & (L)ive load