Design for Strength +...

- **strength design**
  - forces & material

- **serviceability**
  - limit deflection and cracking
  - control noise & vibration
  - no excessive settlement of foundations
  - durability
  - appearance
  - component damage
  - ponding
Beam Deformations

- curvature relates to
  - bending moment
  - modulus of elasticity
  - moment of inertia

\[
\frac{1}{R} = \frac{M}{EI}
\]

\[
curvature = \frac{M(x)}{EI}
\]

\[
\theta = \text{slope} = \int \frac{M(x)}{EI} \, dx
\]

\[
\Delta = \text{deflection} = \int \int \frac{M(x)}{EI} \, dx
\]
Deflected Shape & $M(x)$

- $-M(x)$ gives shape indication
- boundary conditions must be met
Boundary Conditions

- at pins, rollers, fixed supports: $y = 0$
- at fixed supports: $\theta = 0$
- at inflection points from symmetry: $\theta = 0$
- $y_{\text{max}}$ at $\frac{dy}{dx} = 0$
Superpositioning

- if w can be superpositioned
  - $\theta$ & $y$ can
  - elastic range only!
**Deflection Limits**

- based on service condition, severity

<table>
<thead>
<tr>
<th>Use</th>
<th>LL only</th>
<th>DL+LL</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Roof beams:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industrial</td>
<td>L/180</td>
<td>L/120</td>
</tr>
<tr>
<td>Commercial</td>
<td></td>
<td></td>
</tr>
<tr>
<td>plaster ceiling</td>
<td>L/240</td>
<td>L/180</td>
</tr>
<tr>
<td>no plaster</td>
<td>L/360</td>
<td>L/240</td>
</tr>
<tr>
<td><strong>Floor beams:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ordinary Usage</td>
<td>L/360</td>
<td>L/240</td>
</tr>
<tr>
<td>Roof or floor (damageable elements)</td>
<td>L/480</td>
<td></td>
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</tbody>
</table>
Lateral Buckling

• lateral buckling caused by compressive forces at top coupled with insufficient rigidity
• can occur at low stress levels
• stiffen, brace or bigger $I_y$
Local Buckling

- steel I beams
- flange
  - buckle in direction of smaller radius of gyration
- web
  - force
  - “crippling”
Local Buckling

- flange
- web
Shear in Web

- panels in plate girders or webs with large shear
- buckling in compression direction
- add stiffeners

(a) Shear Failure
(b) Shear Buckling
Shear in Web

- plate girders and stiffeners
Beam Design

1. **Know** $F_{all}$ **for the material or**
   $F_U$ **for LRFD**

2. **Draw** $V$ & $M$, **finding** $M_{max}$

3. **Calculate** $S_{req'd}$
   \[
   S_{req'd} = \left(f_b \leq F_b\right)
   \]

4. **Determine** section size
   \[
   S = \frac{bh^2}{6}
   \]
Beam Design

4*. Include self weight for $M_{\text{max}}$
   - and repeat 3 & 4 if necessary

5. Consider lateral stability

Unbraced roof trusses were blown down in 1999 at this project in Moscow, Idaho.

Photo: Ken Carper
Beam Design

6. Evaluate shear stresses - horizontal

- \( f_v \leq F_v \)
- \( W \) and rectangles
  \[
  f_{v_{\text{max}}} = \frac{3V}{2A} \approx \frac{V}{A_{\text{web}}}
  \]
- thin walled sections
  \[
  f_{v_{\text{max}}} = \frac{VQ}{Ib}
  \]
Beam Design

7. Provide adequate bearing area at supports

\[ f_p = \frac{P}{A} \leq F_p \]
Beam Design

8. Evaluate torsion

\[ f_v \leq F_v \]

- circular cross section

\[ f_v = \frac{T \rho}{J} \]

- rectangular

\[ f_v = \frac{T}{c_1 ab^2} \]

<table>
<thead>
<tr>
<th>a/b</th>
<th>c_1</th>
<th>c_2</th>
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<tr>
<td>\infty</td>
<td>0.333</td>
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</tbody>
</table>
Beam Design

9. Evaluate deflections

\[ y_{\text{max}}(x) = \Delta_{\text{actual}} \leq \Delta_{\text{allowable}} \]
Beam Design

9. how to read charts

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD

Total Equiv. Uniform Load . . . . = \( w l \)

\[ R = V \quad . . . . . . . . . . . . = \frac{wl}{2} \]

\[ V_x \quad . . . . . . . . . . . . = w\left(\frac{l}{2} - x\right) \]

\[ M_{\text{max. }} \quad \text{(at center)} \quad . . . . . = \frac{wl^2}{8} \]

\[ M_x \quad . . . . . . . . . . . . = \frac{wx}{2} \left(l - x\right) \]

\[ \Delta_{\text{max.}} \quad \text{(at center)} \quad . . . . . = \frac{5wl^4}{384EI} \]

\[ \Delta_x \quad . . . . . . . . . . . . = \frac{wx}{24EI} \left(l^3 - 2lx^2 + x^3\right) \]