Moments

- forces have the tendency to make a body rotate about an axis

- same translation but different rotation
Moments

(a) Unloaded.

(b) Loaded.

Figure 2.33  Moment on a cantilever beam.

Figure 2.34  An example of torsion on a cantilever beam.
Moments

- A force acting at a different point causes a different moment:
Moments

- **defined by magnitude and direction**
- **units**: $N \cdot m$, $k \cdot ft$
- **direction**:
  - $+ \text{ ccw (right hand rule)}$
  - $- \text{ cw}$
- **value found from $F$ and $\perp$ distance**

\[ M = F \cdot d \]

- $d$ also called “lever” or “moment” arm
Moments

- with same $F$:

$$M_A = F \cdot d_1 < M_A = F \cdot d_2$$

(bigger)
Moments

- additive with sign convention
- can still move the force along the line of action
Moments

- Varignon’s Theorem
  - resolve a force into components at a point and finding perpendicular distances
  - calculate sum of moments
  - equivalent to original moment
- makes life easier!
  - geometry
  - when component runs through point, d=0
Physics & Moments of a Force

- moments of a force
  - introduced in Physics as “Torque Acting on a Particle”
  - and used to satisfy rotational equilibrium
Physics and Moments of a Force

- my Physics book:
Moment Couples

- 2 forces
  - same size
  - opposite direction
  - distance $d$ apart
  - cw or ccw

$$M = F \cdot d$$

- not dependant on point of application

$$M = F \cdot d_1 - F \cdot d_2$$
Moment Couples

• equivalent couples
  – same magnitude and direction
  – $F$ & $d$ may be different
Moment Couples

- *added just like moments caused by one force*
- *can replace two couples with a single couple*

\[
\begin{align*}
300 \text{ N} & \quad + \quad 200 \text{ N} \quad = \quad 240 \text{ N} \\
100 \text{ mm} & \quad + \quad 150 \text{ mm} \quad = \quad 250 \text{ mm}
\end{align*}
\]
Moment Couples

- moment couples in structures

The flanges of a steel beam are welded to the flange of a column. Equal and opposite forces $T$ and $C$ in the beam flanges form a couple with moment $M$ that is transferred into the column.
Equivalent Force Systems

• two forces at a point is equivalent to the resultant at a point
• resultant is equivalent to two components at a point
• resultant of equal & opposite forces at a point is zero
• put equal & opposite forces at a point (sum to 0)
• transmission of a force along action line
Force-Moment Systems

- single force causing a moment can be replaced by the same force at a different point by providing the moment that force caused

- moments are shown as arched arrows
Force-Moment Systems

- a force-moment pair can be replaced by a force at another point causing the original moment

\[ F \cdot d = M \]

\[ F_{A'} \cdot d_{A'} = F_A \cdot d_A \]
Parallel Force Systems

- forces are in the same direction
- can find resultant force
- need to find location for equivalent moments

\[ A \cdot a \]

\[ B \cdot b \]

\[ (A + B) \cdot x \]