centers of gravity - centroids

Center of Gravity
- "average" x & y from moment

\[ \sum M_y = \sum_{i=1}^{n} x_i \Delta W_i = \bar{x} W \quad \Rightarrow \quad \bar{x} = \frac{\sum (x \Delta W)}{W} \]

\[ \sum M_x = \sum_{i=1}^{n} y_i \Delta W_i = \bar{y} W \quad \Rightarrow \quad \bar{y} = \frac{\sum (y \Delta W)}{W} \]

Center of Gravity
- location of equivalent weight
- determined with calculus

\[ W = \int dW \]

Centroid
- "average" x & y of an area
- for a volume of constant thickness
  - \( \Delta W = \gamma \Delta A \) where \( \gamma \) is weight/volume
  - center of gravity = centroid of area

\[ \bar{x} = \frac{\sum (x \Delta A)}{A} \]

\[ \bar{y} = \frac{\sum (y \Delta A)}{A} \]
Centroid
- for a line, sum up length
\[ \bar{x} = \frac{\sum(x\Delta L)}{L} \]
\[ \bar{y} = \frac{\sum(y\Delta L)}{L} \]

1st Moment Area
- math concept
- the moment of an area about an axis
\[ Q_x = \bar{y}A \]
\[ Q_y = \bar{x}A \]

Symmetric Areas
- symmetric about an axis
- symmetric about a center point
- mirrored symmetry

Composite Areas
- made up of basic shapes
- areas can be negative
- (centroids can be negative for any area)
**Basic Procedure**

1. **Draw reference origin (if not given)**
2. **Divide into basic shapes (+/-)**
3. **Label shapes**
4. **Draw table**
5. **Fill in table**
6. **Sum necessary columns**
7. **Calculate \( \bar{x} \) and \( \bar{y} \)**

**Area Centroids**

- **Table 7.1 – pg. 242**

![Table 7.1 - Centroids of Common Shapes of Areas and Lines](image)

- Triangular area
  - \( x = \frac{b}{3} \)
  - \( y = \frac{h}{3} \)
- Quarter-circular area
  - \( x = \frac{sr}{3\pi} \)
  - \( y = \frac{sr}{2\pi} \)
- Semicircular area
  - \( x = 0 \)
  - \( y = \frac{sr}{\pi} \)
- Semi-parabolic area
  - \( x = \frac{3w}{8} \)
  - \( y = \frac{3h}{8} \)
- Parabolic area
  - \( x = 0 \)
  - \( y = \frac{3h}{5} \)