Moments of Inertia

- 2nd moment area
  - math concept
  - area x (distance)^2
- need for behavior of
  - beams
  - columns

Moment of Inertia

- about any reference axis
- can be negative

\[ I_y = \int x^2 \, dA \]
\[ I_x = \int y^2 \, dA \]

- resistance to bending and buckling
Polar Moment of Inertia

- for round-ish shapes
- uses polar coordinates (r and θ)
- resistance to twisting

\[ J_o = \int r^2 dA \]

Radius of Gyration

- measure of inertia with respect to area

\[ r_x = \sqrt{\frac{I_x}{A}} \]

Parallel Axis Theorem

- can find composite \( I \) once composite centroid is known (basic shapes)

\[ I_x = I_{cx} + Ad^2 \]
\[ = \bar{I}_x + Ad^2 \]

\[ I = \sum \bar{I} + \sum Ad^2 \]
\[ \bar{I} = I - Ad^2 \]

Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table with \( A, \bar{x}, \bar{A}, \bar{y}, \bar{y}A, \bar{I}'s, d's, \text{ and } Ad^2's \)
5. Fill in table and get \( \hat{x} \) and \( \hat{y} \) for composite
6. Sum necessary columns
7. Sum \( \bar{I}'s \) and \( Ad^2's \)

\[
\begin{align*}
(d_x &= \hat{x} - \bar{x}) \\
(d_y &= \hat{y} - \bar{y})
\end{align*}
\]
Area Moments of Inertia

- Table 7.2 – pg. 252 (bars refer to centroid)
- $x, y$
- $x', y'$
- $C$

<table>
<thead>
<tr>
<th>Shape</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>$I_x = \frac{b h^3}{12}$, $I_y = \frac{h b^3}{12}$, $I_C = \frac{bh^3}{12} + \frac{h b^3}{12}$</td>
</tr>
<tr>
<td>Triangle</td>
<td>$I_x = \frac{h b^2}{12}$, $I_y = \frac{b h^2}{12}$</td>
</tr>
<tr>
<td>Circle</td>
<td>$I_x = I_y = \frac{\pi r^4}{4}$, $I_C = \frac{\pi r^4}{4}$</td>
</tr>
</tbody>
</table>