Torsion & Temp 1
Lecture 17
Architectural Structures I
ENDS 231
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ARCHITECTURAL STRUCTURES I:
STATICS AND STRENGTH OF MATERIALS
ENDS 231
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lecture
seventeen

torsion
& thermal effects

Torsional Stress & Strain
• can see torsional stresses & twisting of axi-symmetrical cross sections
  – torque
  – remain plane
  – undistorted
  – rotates
• not true for square sections....

Shear Stress Distribution
• depend on the deformation
• \( \phi = \) angle of twist
  – measure
• can prove planar section doesn’t distort

Shearing Strain
• related to \( \phi \)

\[ \gamma = \frac{\rho \phi}{L} \]
• \( \rho \) is the radial distance from the centroid to the point under strain
• shear strain varies linearly along the radius: \( \gamma_{\text{max}} \) is at outer diameter
**Torsional Stress - Strain**

- know \( f_v = \tau = G \cdot \gamma \) and \( \gamma = \frac{\rho \phi}{L} \)
- so \( \tau = G \cdot \frac{\rho \phi}{L} \)
- where \( G \) is the Shear Modulus

**Shear Stress**

- \( \tau_{\text{max}} \) happens at outer diameter
- combined shear and axial stresses
  - maximum shear stress at 45° “twisted” plane

**Torsional Stress - Strain**

- from \( T = \Sigma \tau(\rho)\Delta A \)
- can derive \( T = \frac{\tau J}{\rho} \)
  - where \( J \) is the polar moment of inertia
  - elastic range \( \tau = \frac{T \rho}{J} \)

**Shear strain**

- knowing \( \tau = G \cdot \frac{\rho \phi}{L} \) and \( \tau = \frac{T \rho}{J} \)
- solve: \( \phi = \frac{TL}{JG} \)
- composite shafts: \( \phi = \sum \frac{T_i L_i}{J_i G_i} \)
Noncircular Shapes

- torsion depends on J
- plane sections don’t remain plane
- $\tau_{\text{max}}$ is still at outer diameter

$$\tau_{\text{max}} = \frac{T}{c_1 ab^2} \quad \phi = \frac{TL}{c_2 ab^3 G}$$

- where $a$ is longer side ($> b$)

Open Thin-Walled Sections

- with very large $a/b$ ratios:

$$\tau_{\text{max}} = \frac{T}{\frac{1}{3} ab^2} \quad \phi = \frac{TL}{\frac{1}{3} ab^3 G}$$

Shear Flow in Closed Sections

- $q$ is the internal shear force/unit length

$$\tau = \frac{T}{2ta} \quad \phi = \frac{TL}{4ta^2} \sum_i s_i t_i$$

- $a$ is the area bounded by the centerline
- $s_i$ is the length segment, $t_i$ is the thickness

Shear Flow in Open Sections

- each segment has proportion of $T$ with respect to torsional rigidity,

$$\tau_{\text{max}} = \frac{Tt_{\text{max}}}{\frac{1}{3} \sum b_i t_i^3}$$

- total angle of twist:

$$\phi = \frac{TL}{\frac{1}{3} G \sum b_i t_i^3}$$

- I beams - web is thicker, so $\tau_{\text{max}}$ is in web
Deformation Relationships

- **physical movement**
  - axially (same or zero)
  - rotations from axial changes

\[ \delta = \frac{PL}{AE} \]

relates \( \delta \) to \( P \)

Deformations from Temperature

- atomic chemistry reacts to changes in energy
- solid materials
  - can contract with decrease in temperature
  - can expand with increase in temperature
- linear change can be measured per degree

Thermal Deformation

- \( \alpha \) - the rate of strain per degree
- **UNITS**: °F, °C

length change:
\[ \delta_T = \alpha (\Delta T) L \]

thermal strain:
\[ \varepsilon_T = \alpha (\Delta T) \]

- no stress when movement allowed

Coefficients of Thermal Expansion

<table>
<thead>
<tr>
<th>Material</th>
<th>Coefficients (( \alpha )) [in./in./°F]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood</td>
<td>3.0 \times 10^{-6}</td>
</tr>
<tr>
<td>Glass</td>
<td>4.4 \times 10^{-6}</td>
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<tr>
<td>Concrete</td>
<td>5.5 \times 10^{-6}</td>
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<tr>
<td>Cast Iron</td>
<td>5.9 \times 10^{-6}</td>
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<tr>
<td>Steel</td>
<td>6.5 \times 10^{-6}</td>
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<tr>
<td>Wrought Iron</td>
<td>6.7 \times 10^{-6}</td>
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<tr>
<td>Copper</td>
<td>9.3 \times 10^{-6}</td>
</tr>
<tr>
<td>Bronze</td>
<td>10.1 \times 10^{-6}</td>
</tr>
<tr>
<td>Brass</td>
<td>10.4 \times 10^{-6}</td>
</tr>
<tr>
<td>Aluminum</td>
<td>12.8 \times 10^{-6}</td>
</tr>
</tbody>
</table>
Stresses and Thermal Strains

- if thermal movement is restrained, stresses are induced

1. bar pushes on supports
2. support pushes back
3. reaction causes internal stress \( f = \frac{P}{A} = \frac{\delta E}{L} \)

Superposition Method

- can remove a support to make it look determinant
- replace the support with a reaction
- enforce the geometry constraint

\[
\begin{align*}
\delta_p &= -\frac{PL}{AE} \\
\delta_T &= \alpha(\Delta T)L \\
\text{sub:} & \quad -\frac{PL}{AE} + \alpha(\Delta T)L = 0 \\

f &= -\frac{P}{A} = -\alpha(\Delta T)E
\end{align*}
\]