lecture eleven

centers of gravity- centroids
Center of Gravity

- location of equivalent weight
- determined with calculus

- sum element weights \[ W = \int dW \]
Center of Gravity

- “average” x & y from moment

\[
\sum M_y = \sum_{i=1}^{n} x_i \Delta W_i = \bar{x}W \implies \bar{x} = \frac{\sum (x \Delta W)}{W}
\]

“bar” means average

\[
\sum M_x = \sum_{i=1}^{n} y_i \Delta W_i = \bar{y}W \implies \bar{y} = \frac{\sum (y \Delta W)}{W}
\]
Centroid

- “average” x & y of an area
- for a volume of constant thickness
  - \( \Delta W = \gamma t \Delta A \) where \( \gamma \) is weight/volume
  - center of gravity = centroid of area

\[
\bar{x} = \frac{\sum (x \Delta A)}{A}
\]

\[
\bar{y} = \frac{\sum (y \Delta A)}{A}
\]
Centroid

- for a line, sum up length

\[ x = \frac{\sum(x\Delta L)}{L} \]
\[ y = \frac{\sum(y\Delta L)}{L} \]
1st Moment Area

• math concept
• the moment of an area about an axis

\[ Q_x = \bar{y}A \]
\[ Q_y = \bar{x}A \]
Symmetric Areas

• symmetric about an axis

• symmetric about a center point

• mirrored symmetry
Composite Areas

• made up of basic shapes
• areas can be negative
• (centroids can be negative for any area)
Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table
5. Fill in table
6. Sum necessary columns
7. Calculate $\hat{x}$ and $\hat{y}$
### Area Centroids

- **Table 7.1 – pg. 242**

<table>
<thead>
<tr>
<th>Shape</th>
<th>$\bar{x}$</th>
<th>$\bar{y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular area</td>
<td>$\frac{b}{3}$</td>
<td>$\frac{h}{3}$</td>
</tr>
<tr>
<td>Quarter-circular area</td>
<td>$\frac{4r}{3\pi}$</td>
<td>$\frac{4r}{3\pi}$</td>
</tr>
<tr>
<td>Semicircular area</td>
<td>$0$</td>
<td>$\frac{4r}{3\pi}$</td>
</tr>
<tr>
<td>Semiparabolic area</td>
<td>$\frac{3a}{8}$</td>
<td>$\frac{3h}{5}$</td>
</tr>
<tr>
<td>Parabolic area</td>
<td>$0$</td>
<td>$\frac{3h}{5}$</td>
</tr>
</tbody>
</table>

*Centroids of Common Shapes of Areas and Lines*