moment of inertia of an area
Moments of Inertia

• 2\textsuperscript{nd} moment area
  – math concept
  – area \times (distance)^2

• need for behavior of
  – beams
  – columns
Moment of Inertia

• about any reference axis
• can be negative

\[ I_y = \int x^2 \, dA \]
\[ I_x = \int y^2 \, dA \]

• resistance to bending and buckling
Moment of Inertia

• larger area away for same distance
  – larger $I$
Polar Moment of Inertia

- for round-ish shapes
- uses polar coordinates (r and θ)
- resistance to twisting

\[ J_o = \int r^2 \, dA \]
Radius of Gyration

- measure of inertia with respect to area

\[ r_x = \sqrt{\frac{I_x}{A}} \]
Parallel Axis Theorem

- can find composite $I$ once composite centroid is known (basic shapes)

\[ I_x = I_{cx} + Ad_y^2 \]
\[ = \overline{I}_x + Ad_y^2 \]

\[ I = \sum \overline{I} + \sum Ad^2 \]

\[ \overline{I} = I - Ad^2 \]
Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table with $A$, $\bar{x}$, $\bar{x}A$, $\bar{y}$, $\bar{y}A$, $\bar{I}$’s, $d$’s, and $Ad^2$’s
5. Fill in table and get $\hat{x}$ and $\hat{y}$ for composite
6. Sum necessary columns
7. Sum $\bar{I}$’s and $Ad^2$’s

\[
(d_x = \hat{x} - \bar{x}) \\
(d_y = \hat{y} - \bar{y})
\]
Area Moments of Inertia

- Table 7.2 – pg. 252: (bars refer to centroid)
  - $x$, $y$
  - $x'$, $y'$
  - $C$

- Rectangle
  - $I_x = \frac{1}{12}bh^3$
  - $I_y = \frac{1}{12}b'h^3$
  - $I_c = \frac{1}{2}bh(b^2 + h^2)$

- Triangle
  - $I_x = \frac{1}{24}bh^3$
  - $I_c = \frac{1}{2}bh^3$

- Circle
  - $I_x = I_y = \frac{1}{4\pi}r^4$
  - $J_o = \frac{1}{2\pi}r^4$