elasticity
& strain
Deformations

- materials deform
- axially loaded materials change length
- normal stress is load per unit area
- STRAIN:
  - change in length over length
  - UNITLESS

\[ \varepsilon = \frac{\delta}{L} \]
Shearing Strain

- deformations with shear
- parallelogram
- change in angles
- stress: \( \tau \)
- strain: \( \gamma \)
  - unitless (radians)

\[
\gamma = \frac{\delta_s}{L} = \tan \phi \cong \phi
\]
Shearing Strain

- deformations with torsion
- twist
- change in angle of line

- stress: \( \tau \)
- strain: \( \gamma = \frac{\rho \phi}{L} \)
- unitless (radians)
Load and Deformation

- for stress, need $P$ & $A$
- for strain, need $\delta$ & $L$
  - how?
  - TEST with load and measure
  - plot $P/A$ vs. $\varepsilon$
Material Behavior

• every material has its own response
  – 10,000 psi
  – $L = 10\text{ in}$
  – Douglas Fir vs. steel?

Figure 5.20  Stress-strain diagram for various materials.
Behavior Types

- ductile - “necking”
- true stress
  \[ f = \frac{P}{A} \]

- engineering stress
  – (simplified)
  \[ f = \frac{P}{A_o} \]
Behavior Types

- **brittle**

- **semi-brittle**
Stress to Strain

• important to us in $f$-$\varepsilon$ diagrams:
  – straight section
  – LINEAR-ELASTIC
  – recovers shape (no permanent deformation)

Figure 5.20  Stress-strain diagram for various materials.
Hooke’s Law

- **straight line has constant slope**
- **Hooke’s Law**
  \[ f = E \cdot \varepsilon \]
- **E**
  - Modulus of elasticity
  - Young’s modulus
  - units just like stress
Stiffness

- ability to resist strain

- steels
  - same $E$
  - different yield points
  - different ultimate strength

Figure 5.20  Stress-strain diagram for various materials.
Isotropy & Anisotropy

- **ISOTROPIC**
  - materials with $E$ same at any direction of loading
  - ex. steel

- **ANISOTROPIC**
  - materials with different $E$ at any direction of loading
  - ex. wood
Elastic, Plastic, Fatigue

- elastic springs back
- plastic has permanent deformation
- fatigue caused by reversed loading cycles
Plastic Behavior

- ductile

Figure 5.22  Stress-strain diagram for mild steel (A36) with key points highlighted.
Lateral Strain

- or “what happens to the cross section with axial stress”

\[ \varepsilon_x = \frac{f_x}{E} \]

\[ f_y = f_z = 0 \]

- strain in lateral direction
  - negative
  - equal for isometric materials

\[ \varepsilon_y = \varepsilon_z \]
Poisson’s Ratio

- constant relationship between longitudinal strain and lateral strain

\[
\mu = - \frac{\text{lateral strain}}{\text{axial strain}} = - \frac{\varepsilon_y}{\varepsilon_x} = - \frac{\varepsilon_z}{\varepsilon_x}
\]

\[
\varepsilon_y = \varepsilon_z = - \frac{\mu f_x}{E}
\]

- sign! \(0 < \mu < 0.5\)
Calculating Strain

- from Hooke’s law
  \[ f = E \cdot \varepsilon \]

- substitute
  \[ \frac{P}{A} = E \cdot \frac{\delta}{L} \]

- get
  \[ \delta = \frac{PL}{AE} \]
Orthotropic Materials

- non-isometric
- directional values of $E$ and $\mu$
- ex:
  - plywood
  - laminates
  - polymer composites
Stress Concentrations

- **why we use** $f_{\text{ave}}$
- **increase in stress at changes in geometry**
  - sharp notches
  - holes
  - corners

---

Figure 5.35  Stress trajectories around a hole.
Maximum Stresses

- If we need to know where \( \max f \) and \( f_v \) happen:

\[
\begin{align*}
\theta &= 0^\circ \rightarrow \cos \theta = 1 \\
\theta &= 45^\circ \rightarrow \cos \theta = \sin \theta = \sqrt{0.5}
\end{align*}
\]

\[
\begin{align*}
f_{\text{max}} &= \frac{P}{A_o} \\
f_{v-\text{max}} &= \frac{P}{2A_o} = \frac{f_{\text{max}}}{2}
\end{align*}
\]
Maximum Stresses

**FIG. 2-37** Shear failure along a 45° plane of a wood block loaded in compression

**FIG. 2-38** Slip bands (or Lüders’ bands) in a polished steel specimen loaded in tension
Design of Members

- beyond allowable stress...
- materials aren’t uniform 100% of the time
  - ultimate strength or capacity to failure may be different and some strengths hard to test for

- **RISK & UNCERTAINTY**

\[ f_u = \frac{P_u}{A} \]
Factor of Safety

• accommodate uncertainty with a safety factor:

\[ \text{allowable load} = \frac{\text{ultimate load}}{F.S} \]

• with linear relation between load and stress:

\[ F.S = \frac{\text{ultimate load}}{\text{allowable load}} = \frac{\text{ultimate stress}}{\text{allowable stress}} \]
Load and Resistance Factor Design

• loads on structures are
  – not constant
  – can be more influential on failure
  – happen more or less often
  – UNCERTAINTY

\[ R_u = \gamma_D R_D + \gamma_L R_L \leq \phi R_n \]

\( \phi \) - resistance factor
\( \gamma \) - load factor for (D)ead & (L)ive load