stability and columns
Additional Design Criteria

• designed for strength & stresses
• designed for serviceability & deflection
• need to design for stability
  – ability to support a specified load without sudden or unacceptable deformations
Column Behavior

• objects like lowest energy state
Stable Equilibrium

- energy added
- things don’t change
Neutral Equilibrium

- energy added
- things change, but not much

\[ P = P_{\text{crit}} = \frac{\pi^2 EI}{L^2} \]
Unstable Equilibrium

- energy added
- things change drastically
Column Buckling

- axially loaded columns
- long & slender
  - unstable equilibrium = buckling
  - sudden and not good
Modeling

- can be modeled with a spring at mid-height
- when moment from deflection exceeds the spring capacity ... “boing”
- critical load $P$
Effect of Length

- long & slender
- short & stubby
Buckling Load

- related to deflected shape \((P\Delta)\)
- shape of sine wave
- Euler’s Formula
- \(I\) minimum

\[
P_{\text{critical}} = \frac{\pi^2 EI_{\text{min}}}{(L)^2}
\]
Critical Stress

- short columns
  
  \[ f_{\text{critical}} = \frac{P_{\text{actual}}}{A} < F_a \]

- slenderness ratio = \( \frac{L_e}{r} \) (L/d)

- radius of gyration = \( r = \sqrt{\frac{I}{A}} \)

\[
 f_{\text{critical}} = \frac{P_{\text{critical}}}{A} = \frac{\pi^2 E Ar^2}{A (L_e)^2} = \frac{\pi^2 E}{\left(\frac{L_e}{r}\right)^2}
\]

\[
 P_{\text{critical}} = \frac{\pi^2 EA}{\left(\frac{L_e}{r}\right)^2}
\]
Critical Stresses

- when a column gets stubby, $F_y$ will limit the load
- real world has loads with eccentricity
- $C_c$ for steel and allowable stress

$$L_e/r > C_c = \sqrt{\frac{2\pi^2 E}{F_y}}$$

Fig. 10.9
Effective Length

- end conditions affect shape
- effective length factor, $K \quad L_e = K \cdot L$
**Bracing**

- bracing affects shape of buckle in one direction
- both should be checked!

(a) No bracing.  
(b) Braced at midpoint.  
(c) Third-point bracing.  
(d) Asymmetric bracing.