ARCHITECTURAL STRUCTURES I:
STATICS AND STRENGTH OF MATERIALS
ENDS 231
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lecture twenty five

eccentric loading:
beam-columns
Centric & Eccentric Loading

- **centric**
  - allowable stress from strength or buckling
- **eccentric**
  - combined stresses
Eccentric Loading

- axial + bending

\[ f_{\text{max}} = \frac{P}{A} + \frac{Mc}{I} \]

\[ M = P \cdot e \]

- design

\[ f_{\text{max}} \leq F_{cr} = \frac{f_{cr}}{F.S.} \]
Eccentric Loading

- find $e$ such that the minimum stress = 0

$$f_{\min} = \frac{P}{A} - \frac{(Pe)c}{I} = 0$$

- area defined by $e$ from centroid is the kern
Eccentric Loading

– when there is eccentricity in two directions

\[ M_1 = P \cdot e_1 \quad M_2 = P \cdot e_2 \]

\[ f_{\text{max}} = \frac{P}{A} + \frac{M_1 y}{I} + \frac{M_2 z}{I} \]

– biaxial bending
Stress Limit Conditions

– ASD interaction formula

\[
\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0
\]

– with biaxial bending

\[
\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0
\]
Stress Limit Conditions

– in reality, as the column flexes, the moment increases

– **P-Δ effect**

\[
\frac{f_a}{F_a} + \frac{f_b \times (\text{Magnification factor})}{F_{bx}} \leq 1.0
\]
Design

- **satisfy**
  - strength
  - stability
- **pick**
  - section
Design

- ASD Steel

\[
\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{1 - \frac{f_a}{F'_e}} F_{bx} + \frac{C_{my} f_{by}}{1 - \frac{f_a}{F'_e}} F_{by} \leq 1.0
\]

*\( C_m \) – modification factor for end conditions

\[ = 0.6 - 0.4(M_1/M_2) \text{ or } 0.85 \text{ restrained} \]

*\( F'_e \) – allowable buckling strength

*\( () \) term – magnification factor for P-\( \Delta \)
Design

• Wood

\[
\left( \frac{f_c}{F_c'} \right)^2 + \frac{f_{bx}}{F_{bx}' \left[ 1 - \frac{f_c}{F_{cEx}} \right]} \leq 1.0
\]

() term – magnification factor for P-\(\Delta\)

\(F'_{bx}\) – allowable bending strength
Design

- **LRFD Steel**

- for \( \frac{P_u}{\phi_c P_n} \geq 0.2 \):
  \[
  \frac{P_u}{\phi_c P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) \leq 1.0
  \]

- for \( \frac{P_u}{\phi_c P_n} < 0.2 \):
  \[
  \frac{P_u}{2\phi_c P_n} + \left( \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) \leq 1.0
  \]

\( \phi_c \) - resistance factor for compression = 0.85

\( \phi_b \) - resistance factor for bending = 0.9
Design Steps Knowing Loads

1. assume limiting stress
   - buckling, axial stress, combined stress

2. solve for r, A or S

3. pick trial section

4. analyze stresses

5. section ok?

6. stop when section is ok