Deformations

• materials deform
• axially loaded materials change length
• normal stress is load per unit area
• STRAIN:
  – change in length over length
  – UNITLESS
  \[ \varepsilon = \frac{\delta}{L} \]

Shearing Strain

• deformations with shear
• parallelogram
• change in angles
• stress: \( \tau \)
• strain: \( \gamma = \frac{\delta}{L} = \tan \phi \cong \phi \)
  – unitless (radians)
Load and Deformation

- for stress, need P & A
- for strain, need δ & L
  - how?
  - TEST with load and measure
  - plot P/A vs. ε

Material Behavior

- every material has its own response
  - 10,000 psi
  - L = 10 in
  - Douglas Fir vs. steel?

Behavior Types

- ductile - “necking”
- true stress
  \[ f = \frac{P}{A} \]
- engineering stress
  - (simplified)
  \[ f = \frac{P}{A_0} \]

Figure 5.20  Stress-strain diagram for various materials.

Behavior Types

- brittle
- semi-brittle

Figure 2.11  Stress-strain diagram for a typical brittle material.

Figure 2.14  Stress-strain diagram for concrete.
Stress to Strain

- important to us in $f$-$\varepsilon$ diagrams:
  - straight section
  - LINEAR-ELASTIC
  - recovers shape (no permanent deformation)

Hooke’s Law

- straight line has constant slope
- Hooke’s Law
  \[ f = E \cdot \varepsilon \]
- $E$
  - Modulus of elasticity
  - Young’s modulus
  - units just like stress

Stiffness

- ability to resist strain

- steels
  - same $E$
  - different yield points
  - different ultimate strength

Isotropy & Anisotropy

- ISOTROPIC
  - materials with $E$ same at any direction of loading
  - ex. steel

- ANISOTROPIC
  - materials with different $E$ at any direction of loading
  - ex. wood
**Elastic, Plastic, Fatigue**

- elastic springs back
- plastic has permanent deformation
- fatigue caused by reversed loading cycles

**Plastic Behavior**

- ductile

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**Lateral Strain**

- or “what happens to the cross section with axial stress”

\[ \varepsilon_x = \frac{f_x}{E} \]

\[ f_y = f_z = 0 \]

- strain in lateral direction
  - negative
  - equal for isometric materials

\[ \varepsilon_y = \varepsilon_z \]

**Poisson’s Ratio**

- constant relationship between longitudinal strain and lateral strain

\[ \mu = \frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x} \]

\[ \varepsilon_y = \varepsilon_z = -\frac{\mu f_x}{E} \]

- sign!

\[ 0 < \mu < 0.5 \]
Calculating Strain

- from Hooke’s law
  \[ f = E \cdot \varepsilon \]
- substitute
  \[ \frac{P}{A} = E \cdot \frac{\delta}{L} \]
- get \( \Rightarrow \)
  \[ \delta = \frac{PL}{AE} \]

Orthotropic Materials

- non-isometric
- directional values of \( E \) and \( \mu \)
- ex:
  - plywood
  - laminates
  - polymer composites

Stress Concentrations

- why we use \( f_{ave} \)
- increase in stress at changes in geometry
  - sharp notches
  - holes
  - corners

Maximum Stresses

- if we need to know where \( f_{max} \) and \( f_v \) happen:
  \[ \theta = 0^\circ \rightarrow \cos \theta = 1 \]
  \[ f_{max} = \frac{P}{A_o} \]
  \[ \theta = 45^\circ \rightarrow \cos \theta = \sin \theta = \sqrt{0.5} \]
  \[ f_{v\text{-max}} = \frac{P}{2A_o} = \frac{f_{max}}{2} \]
Maximum Stresses

Design of Members

- beyond allowable stress...
- materials aren’t uniform 100% of the time
  - ultimate strength or capacity to failure may be different and some strengths hard to test for
- RISK & UNCERTAINTY

\[ f_u = \frac{P_u}{A} \]

Factor of Safety

- accommodate uncertainty with a safety factor:
  \[ \text{allowable load} = \frac{\text{ultimate load}}{F.S} \]
- with linear relation between load and stress:
  \[ F.S = \frac{\text{ultimate load}}{\text{allowable load}} = \frac{\text{ultimate stress}}{\text{allowable stress}} \]

Load and Resistance Factor Design

- loads on structures are
  - not constant
  - can be more influential on failure
  - happen more or less often
- UNCERTAINTY

\[ R_u = \gamma_D R_D + \gamma_L R_L \leq \phi R_n \]

\( \phi \) - resistance factor
\( \gamma \) - load factor for (D)ead & (L)ive load