Steel & Wood Column Design

Design Aims

If we know the loads, we can select a section that is adequate for strength & buckling.

If we know the length, we can find the limiting load satisfying strength & buckling.

Design Code Methodologies

Allowable Stress Design (ASD): the stress in a member must be less than an allowable stress which is equal to the yield stress divided by a factor of safety.

Load and Resistance Factor Design: more efficient method that factors loads for importance and compares the summation to a nominal strength that has been adjusted by a reduction factor.

Allowable Stress Design - Steel

American Institute of Steel Construction (AISC) Manual of ASD, 9th ed:

Long and slender: \( [L_e/r \geq C_c, \text{ preferably } < 200] \)

\[
F_{\text{allowable}} = \frac{F_{cr}}{F.S.} = \frac{12\pi^2 E}{23(KL/r)^2}
\]

The yield limit is idealized into a parabolic curve that blends into the Euler’s Formula at \( C_c \).

With \( F_y = 36 \text{ ksi}, C_c = 126.1 \)

With \( F_y = 50 \text{ ksi}, C_c = 107.0 \)

Short and stubby: \( [L_e/r < C_c] \)

\[
F_a = \left[ 1 - \left( \frac{KL/r}{2C_c^2} \right)^2 \right] \frac{F_y}{F.S.}
\]

with:

\[
F.S. = \frac{5}{3} + \frac{3(KL/r)^2}{8C_c} - \frac{(KL/r)^3}{8C_c^3}
\]
Procedure for Analysis

1. Calculate KL/r for each axis (if necessary). The largest will govern the buckling load.
2. Find $F_a$ as a function of KL/r from Table 10.1 or 10.2 (pp. 361-364)
3. Compute $P_{allowable} = F_a \cdot A$ or alternatively compute $f_{actual} = \frac{P}{A}$
4. Is the design satisfactory?
   - Is $P \leq P_{allowable}$? $\Rightarrow$ yes, it is; no, it is no good
   - or Is $f_{actual} \leq F_a$? $\Rightarrow$ yes, it is; no, it is no good

Procedure for Design

1. Guess a size by picking a section.
2. Calculate KL/r for each axis (if necessary). The largest will govern the buckling load.
3. Find $F_a$ as a function of KL/r from Table 10.1 or 10.2 (pp. 361-364)
4. Compute $P_{allowable} = F_a \cdot A$ or alternatively compute $f_{actual} = \frac{P}{A}$
5. Is the design satisfactory?
   - Is $P \leq P_{allowable}$? $\Rightarrow$ yes, it is; no, pick a bigger section and go back to step 2.
   - or Is $f_{actual} \leq F_a$? $\Rightarrow$ yes, it is; no, pick a bigger section and go back to step 2.
6. Check design efficiency by calculating percentage of stress used $= \frac{P_{actual}}{P_{allowable}} \cdot 100\%$
   - If value is between 90-100%, it is efficient.
   - If values is less than 90%, pick a smaller section and go back to step 2.

The critical load with respect to the slenderness ratio is presented in chart format in ASD, 8th ed, as well as the allowable stress charts for compression members.

Allowable Stress Design - Wood


Any slenderness ratio, $L_e/d \leq 50$:

$$f_c = \frac{P}{A} \leq F'_c$$

$$F'_c = F_c \left( C_D \right) \left( C_M \right) \left( C_I \right) \left( C_F \right) \left( C_P \right)$$

The curve uses factors to replicate the combination curve:
where:
\[ F_c' = \text{allowable compressive stress parallel to the grain} \]
\[ F_c = \text{compressive strength parallel to the grain} \]
\[ C_D = \text{load duration factor} \]
\[ C_M = \text{wet service factor (1.0 for dry)} \]
\[ C_t = \text{temperature factor} \]
\[ C_F = \text{size factor} \]
\[ C_p = \text{column stability factor off chart} \]

For preliminary column design:
\[ F_c' = F_c^* C_p = (F_c C_D) C_p \]

Procedure for Analysis

1. Calculate \( L_e/d_{\min} \)
2. Obtain \( F_c' \)
   
   \[ \text{compute } F_{cE} = \frac{K_{cE} E}{(\ell/d)^2} \quad \text{with } K_{cE} = 0.3 \text{ for sawn, } = 0.418 \text{ for glu-lam} \]
3. Compute \( F_c^* \equiv F_c C_D \) with \( C_D = 1 \), normal, \( C_D = 1.25 \) for 7 day roof...
4. Calculate \( F_{cE}/F_c^* \) and get \( C_p \) from Appendix A, Table 14 (pp. 413-414)
5. Calculate \( F_c' = F_c^* C_p \)
6. Compute \( P_{\text{allowable}} = F_c' A \) or alternatively compute \( f_{\text{actual}} = P/A \)
7. Is the design satisfactory?
   
   Is \( P \leq P_{\text{allowable}}? \Rightarrow \) yes, it is; no, it is no good
   
   or Is \( f_{\text{actual}} \leq F_c' \)? \Rightarrow yes, it is; no, it is no good

Procedure for Design

1. Guess a size by picking a section
2. Calculate \( L_e/d_{\min} \)
3. Obtain \( F_c' \)
   
   \[ \text{compute } F_{cE} = \frac{K_{cE} E}{(\ell/d)^2} \quad \text{with } K_{cE} = 0.3 \text{ for sawn, } = 0.418 \text{ for glu-lam} \]
4. Compute \( F_c^* \equiv F_c C_D \) with \( C_D = 1 \), normal, \( C_D = 1.25 \) for 7 day roof...
5. Calculate \( F_{cE}/F_c^* \) and get \( C_p \) from Appendix A, Table 14 (pp. 413-414)
6. Calculate $F'_c = F'_c C_p$

7. Compute $P_{allowable} = F'_c A$ or alternatively compute $f_{actual} = P/A$

8. Is the design satisfactory?
   
   Is $P \leq P_{allowable}$? $\Rightarrow$ yes, it is; no, pick a bigger section and go back to step 2.
   
   or Is $f_{actual} \leq F'_c$? $\Rightarrow$ yes, it is; no, pick a bigger section and go back to step 2.

Load & Resistance Factor Design

American Institute of Steel Construction (AISC) Manual of LRFD, 3rd ed:

$$\sum \gamma_i Q_i \leq \phi P_n$$

where

- $\gamma$ is a load factor
- $Q$ is a load type
- $\phi$ is a resistance factor
- $P_n$ is the nominal load capacity (strength)

Load combinations, ex:

- 1.4D (D is dead load)
- 1.2D + 1.6L (L is live load)

For compression, $\phi_c = 0.85$ and $P_n = A_g F_{cr}$

where:

- $A_g$ is the cross section area and $F_{cr}$ is the critical stress shown below (in Compact Sections).

Compact Sections

Compact sections are defined as sections with flanges continuously connected to the web or webs and the width-thickness ratios are less than limiting values given in the manual. This is to avoid local buckling of the flange or the web.

Formula parts depend on $\lambda_c = \frac{KL}{r \pi} \sqrt{\frac{F_y}{E}}$ where $\frac{KL}{r} = \frac{L_e}{r}$.

when $\lambda_c \leq 1.5$:

$$F_{cr} = \left(0.658 \lambda_c^2\right) F_y$$

where $F_{cr}$ is the critical stress

when $\lambda_c > 1.5$:

$$F_{cr} = \left[\frac{0.877}{\lambda_c^2}\right] F_y$$
### Sample AISC Table for Allowable Axial Loads (ASD)

<table>
<thead>
<tr>
<th>Column Designation</th>
<th>Allowable Axial Loads in kips</th>
</tr>
</thead>
<tbody>
<tr>
<td>W Shapes</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Sample AISC Table for LRFD Design Strength in Compression

<table>
<thead>
<tr>
<th>Column Designation</th>
<th>Design Strength in Compression, f_{Pcr}, kips</th>
</tr>
</thead>
<tbody>
<tr>
<td>W Shapes</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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*Note: The tables above are examples of how AISC tables might look, but the actual tables will contain more detailed data.*
Example 1 (pg 367)
Example Problem 10.10 (Figure 10.41)

A 24-ft.-tall, A572 grade 50, steel column (W14x82) with an $F_y = 50$ ksi has pins at both ends. Its weak axis is braced at midheight, but the column is free to buckle the full 24 ft. in the strong direction. Determine the safe load capacity for this column.

Figure 10.41  (a) Strong axis buckling.  
(b) Weak axis buckling.

Example 2 (pg 371) + chart method
Example Problem 10.14: Design of Steel Columns (Figure 10.48)

Select the most economical W12 x column 18' in height to support an axial load of 600 kips using A572 grade 50 steel. Assume that the column is hinged at the top but fixed at the base.

ALSO: Select the column using the ASD design charts, and the LRFD charts assuming that the load is a dead load (factor of 1.4)
Example 3 (pg 379)
Example Problem 10.18 (Figures 10.60 and 10.61)

An 18' tall 6x8 Southern pine column supports a roof load (dead load plus a 7-day live load) equal to 16 kips. The weak axis of buckling is braced at a point 9'6" from the bottom support. Determine the adequacy of the column.

\[ F_c = 975 \text{ psi}, \quad E = 1.6 \times 10^6 \text{ psi} \]

Figure 10.61  (a) Strong axis. (b) Weak axis.
Example 4 (pg 381)
Example Problem 10.20:
Design of Wood Columns (Figure 10.66)

A 22'-tall glu-lam column is required to support a roof load (including snow) of 40 kips. Assuming 8\(\frac{3}{4}\)" in one dimension (to match the beam width above), determine the minimum column size if the top and bottom are pin supported.

Select from the following sizes:

\[
\begin{align*}
8\frac{3}{4}'' \times 9'' & \ (A = 78.75 \text{ in}^2) \\
8\frac{3}{4}'' \times 10\frac{1}{2}'' & \ (A = 91.88 \text{ in}^2) \\
8\frac{3}{4}'' \times 12'' & \ (A = 105.00 \text{ in}^2)
\end{align*}
\]