elasticity & strain
Deformations

- materials deform
- axially loaded materials change length
- normal stress is load per unit area
- STRAIN:
  - change in length over length
  - UNITLESS

\[ \varepsilon = \frac{\delta}{L} \]
Shearing Strain

- deformations with shear
- parallelogram
- change in angles
- stress: \( \tau \)
- strain: \( \gamma \)
  - unitless (radians)

\[
\gamma = \frac{\delta_s}{L} = \tan \phi \approx \phi
\]
Shearing Strain

- deformations with torsion
- twist
- change in angle of line
- stress: $\tau$
- strain: $\gamma = \frac{\rho \phi}{L}$
  - unitless (radians)
Load and Deformation

- for stress, need $P$ & $A$
- for strain, need $\delta$ & $L$
  - how?
  - TEST with load and measure
  - plot $P/A$ vs. $\varepsilon$
Material Behavior

• every material has its own response
  – 10,000 psi
  – L = 10 in
  – Douglas Fir vs. steel?
Behavior Types

- ductile - “necking”
- true stress

\[ f = \frac{P}{A} \]

- engineering stress
  – (simplified)

\[ f = \frac{P}{A_0} \]
Behavior Types

- brittle

- semi-brittle
Stress to Strain

• **important to us in** $f$-$\varepsilon$ **diagrams:**
  - straight section
  - LINEAR-ELASTIC
  - recovers shape (no permanent deformation)

*Figure 5.20  Stress-strain diagram for various materials.*
Hooke’s Law

- straight line has constant slope
- Hooke’s Law
  \[ f = E \cdot \varepsilon \]
- \( E \)
  - Modulus of elasticity
  - Young’s modulus
  - units just like stress
Stiffness

- ability to resist strain

- steels
  - same $E$
  - different yield points
  - different ultimate strength

Figure 5.20 Stress-strain diagram for various materials.
Isotropy & Anisotropy

- **ISOTROPIC**
  - materials with \( E \) same at any direction of loading
  - ex. steel

- **ANISOTROPIC**
  - materials with different \( E \) at any direction of loading
  - ex. wood is orthotropic
Elastic, Plastic, Fatigue

- elastic springs back
- plastic has permanent deformation
- fatigue caused by reversed loading cycles
Plastic Behavior

- ductile

Figure 5.22 Stress-strain diagram for mild steel (A36) with key points highlighted.
Lateral Strain

• or “what happens to the cross section with axial stress”

\[ \varepsilon_x = \frac{f_x}{E} \]

\[ f_y = f_z = 0 \]

• strain in lateral direction
  – negative
  – equal for isometric materials

\[ \varepsilon_y = \varepsilon_z \]
Poisson’s Ratio

- constant relationship between longitudinal strain and lateral strain

\[
\mu = -\frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}
\]

\[
\varepsilon_y = \varepsilon_z = -\frac{\mu f_x}{E}
\]

- sign! \[0 < \mu < 0.5\]
Calculating Strain

• from Hooke’s law

\[ f = E \cdot \varepsilon \]

• substitute

\[ \frac{P}{A} = E \cdot \frac{\delta}{L} \]

• get ⇒

\[ \delta = \frac{PL}{AE} \]
Orthotropic Materials

• non-isometric
• directional values of $E$ and $\mu$
• ex:
  – plywood
  – laminates
  – polymer composites
Stress Concentrations

- why we use $f_{ave}$
- increase in stress at changes in geometry
  - sharp notches
  - holes
  - corners

Figure 5.35 Stress trajectories around a hole.
Maximum Stresses

- if we need to know where $\max f$ and $f_v$ happen:

\[ \theta = 0^\circ \rightarrow \cos \theta = 1 \quad f_{\text{max}} = \frac{P}{A_o} \]

\[ \theta = 45^\circ \rightarrow \cos \theta = \sin \theta = \sqrt{0.5} \]

\[ f_{v\text{-max}} = \frac{P}{2A_o} = \frac{f_{\text{max}}}{2} \]
Maximum Stresses

**FIG. 2-37** Shear failure along a 45° plane of a wood block loaded in compression

**FIG. 2-38** Slip bands (or Lüders’ bands) in a polished steel specimen loaded in tension
Design of Members

• beyond allowable stress...

• materials aren’t uniform 100% of the time
  – ultimate strength or capacity to failure may be different and some strengths hard to test for

• RISK & UNCERTAINTY

\[ f_u = \frac{P_u}{A} \]
Factor of Safety

- accommodate uncertainty with a safety factor:
  
  \[
  \text{allowable load} = \frac{\text{ultimate load}}{F.S}
  \]

- with linear relation between load and stress:
  
  \[
  F.S = \frac{\text{ultimate load}}{\text{allowable load}} = \frac{\text{ultimate stress}}{\text{allowable stress}}
  \]
Load and Resistance Factor Design

- loads on structures are
  - not constant
  - can be more influential on failure
  - happen more or less often
  - UNCERTAINTY

\[ R_u = \gamma_D R_D + \gamma_L R_L \leq \phi R_n \]

\( \phi \) - resistance factor
\( \gamma \) - load factor for (D)ead & (L)ive load