Eccentric Loading: Beam-Columns
Centric & Eccentric Loading

- **centric**
  - allowable stress from strength or buckling
- **eccentric**
  - combined stresses

![Diagram of centric and eccentric loading](image)
Eccentric Loading
– axial + bending

\[ f_{\text{max}} = \frac{P}{A} + \frac{Mc}{I} \]

\[ M = P \cdot e \]

– design

\[ f_{\text{max}} \leq F_{cr} = \frac{f_{cr}}{F.S.} \]
Eccentric Loading

– find e such that the minimum stress = 0

\[ f_{\text{min}} = \frac{P}{A} - \frac{(Pe)c}{I} = 0 \]

– area defined by e from centroid is the kern
Eccentric Loading

– when there is eccentricity in two directions

\[ M_1 = P \cdot e_1 \quad M_2 = P \cdot e_2 \]

\[ f_{\text{max}} = \frac{P}{A} + \frac{M_1 y}{I} + \frac{M_2 z}{I} \]

– biaxial bending
Stress Limit Conditions

- ASD interaction formula

\[
\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0
\]

- with biaxial bending

\[
\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0
\]

interaction diagram
Stress Limit Conditions

– in reality, as the column flexes, the moment increases

– **P-Δ effect**

\[
\frac{f_a}{F_a} + \frac{f_b \times (Magnification\ factor)}{F_{bx}} \leq 1.0
\]
Design

- **satisfy**
  - strength
  - stability

- **pick**
  - section
Design

• ASD Steel

\[
\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{\left(1 - \frac{f_a}{F'_{ex}}\right) F_{bx}} + \frac{C_{my} f_{by}}{\left(1 - \frac{f_a}{F'_{ey}}\right) F_{by}} \leq 1.0
\]

C_m – modification factor for end conditions

\[= 0.6 - 0.4(M_1/M_2) \text{ or } 0.85 \text{ restrained}\]

F'_e – allowable buckling strength

() term – magnification factor for P-\Delta
Design

• Wood

\[
\left[ \frac{f_c}{F_c''} \right]^2 + \frac{f_{bx}}{F_{bx}' \left[ 1 - \frac{f_c}{F_{cEx}} \right]} \leq 1.0
\]

[] term – magnification factor for P-\(\Delta\)

\(F'_{bx}\) – allowable bending strength
Design

- **LRFD Steel**

  - for \( \frac{P_u}{\phi_c P_n} \geq 0.2 : \quad \frac{P_u}{\phi_c P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) \leq 1.0 \)

  - for \( \frac{P_u}{\phi_c P_n} < 0.2 : \quad \frac{P_u}{2\phi_c P_n} + \left( \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) \leq 1.0 \)

\( \phi_c \) - resistance factor for compression = 0.85
\( \phi_b \) - resistance factor for bending = 0.9
Design Steps Knowing Loads

1. assume limiting stress
   - buckling, axial stress, combined stress

2. solve for r, A or S

3. pick trial section

4. analyze stresses

5. section ok?

6. stop when section is ok