Moment of Inertia 1
Lecture 11
ENDS 231
Su2006abn

ARCHITECTURAL STRUCTURES I:
STATICS AND STRENGTH OF MATERIALS
ENDS 231
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moment of inertia
of an area
Moments of Inertia

- 2nd moment area
  - math concept
  - area x (distance)^2
- need for behavior of
  - beams
  - columns
Moment of Inertia

- about any reference axis
- can be negative

\[ I_y = \int x^2 \, dA \]
\[ I_x = \int y^2 \, dA \]

- resistance to bending and buckling
Moment of Inertia

- larger area away for same distance
  - larger $I$
Polar Moment of Inertia

- for round-ish shapes
- uses polar coordinates \((r \text{ and } \theta)\)
- resistance to twisting

\[
J_o = \int r^2 \, dA
\]
Radius of Gyration

- measure of inertia with respect to area

\[ r_x = \sqrt{\frac{I_x}{A}} \]
Parallel Axis Theorem

- can find composite $I$ once composite centroid is known (basic shapes)

\[ I_x = I_{cx} + A d_y^2 \]
\[ = \bar{I}_x + A d_y^2 \]

\[ I = \sum \bar{I} + \sum A d^2 \]

\[ \bar{I} = I - A d^2 \]
Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table with $A$, $\bar{x}$, $\bar{x}A$, $\bar{y}$, $\bar{y}A$, $I$’s, $d$’s, and $Ad^2$’s
5. Fill in table and get $\hat{x}$ and $\hat{y}$ for composite
6. Sum necessary columns
7. Sum $I$’s and $Ad^2$’s

$\left( \begin{array}{c} d_x = \hat{x} - \bar{x} \\ d_y = \hat{y} - \bar{y} \end{array} \right)$
**Area Moments of Inertia**

- **Table 7.2 – pg. 252:** (bars refer to centroid)
  - $x$, $y$
  - $x'$, $y'$
  - $C$

<table>
<thead>
<tr>
<th>Shape</th>
<th>Area Moment of Inertia</th>
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</thead>
</table>
| Rectangle | $I_x = \frac{1}{12}bh^3$  
          | $I_y = \frac{1}{12}b'h^3$ |
|          | $I_c = \frac{1}{12}bh(b^2 + h^2)$ |
| Triangle | $I_x = \frac{1}{3}bh^3$   
         | $I_y = \frac{1}{3}b'h^3$  |
| Circle  | $I_x = I_y = \frac{1}{4}\pi r^4$ |
|          | $J_o = \frac{1}{2}\pi r^4$  |