Architectural Structures: Form, Behavior, and Design

Arch 331

Dr. Anne Nichols

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Lecture ten

Beams: Bending and Shear Stress
Beam Bending

- Galileo
  - relationship between stress and depth\(^2\)
- can see
  - top squishing
  - bottom stretching

- what are the stress across the section?
Pure Bending

- bending only
- no shear
- axial normal stresses from bending can be found in
  - homogeneous materials
  - plane of symmetry
  - follow Hooke’s law
Bending Moments

• sign convention:

• size of maximum internal moment will govern our design of the section
Normal Stresses

• geometric fit
  – plane sections remain plane
  – stress varies linearly
Neutral Axis

- stresses vary linearly

- zero stress occurs at the centroid

- neutral axis is line of centroids (n.a.)
Derivation of Stress from Strain

- pure bending = arc shape

\[ L = R \theta \]

\[ L_{outside} = (R + y)\theta \]

\[ \varepsilon = \frac{\delta}{L} = \frac{L_{outside} - L}{L} = \frac{(R + y)\theta - R \theta}{R \theta} = \frac{y}{R} \]
Derivation of Stress

- zero stress at n.a.

\[ f = E\varepsilon = \frac{E_y}{R} \]

\[ f_{\text{max}} = \frac{E c}{R} \]

\[ f = \frac{y}{c} f_{\text{max}} \]
Bending Moment

• resultant moment from stresses = bending moment!

\[ M = \Sigma f_y \Delta A \]

\[ = \sum \frac{y f_{\text{max}}}{c} y \Delta A = \frac{f_{\text{max}}}{c} \Sigma y^2 \Delta A = \frac{f_{\text{max}}}{c} I = f_{\text{max}} S \]
**Bending Stress Relations**

\[
\frac{1}{R} = \frac{M}{EI}
\]

Curvature

\[
f_b = \frac{My}{I}
\]

General bending stress

\[
S = \frac{I}{c}
\]

Section modulus

\[
f_b = \frac{M}{S}
\]

Maximum bending stress

\[
S_{\text{required}} \geq \frac{M}{F_b}
\]

Required section modulus for design
Transverse Loading and Shear

- perpendicular loading
- internal shear
- along with bending moment
Bending vs. Shear in Design

- **bending stresses dominate**

- **shear stresses exist horizontally with shear**

- **no shear stresses with pure bending**
Shear Stresses

- horizontal & vertical
Shear Stresses

- horizontal & vertical
Beam Stresses

- horizontal with bending
Equilibrium

- horizontal force $V$ needed

$$V_{\text{longitudinal}} = \frac{V_T Q}{I} \Delta x$$

- $Q$ is a moment area
Moment of Area

- Q is a moment area with respect to the n.a. of area above or below the horizontal

- $Q_{\text{max}}$ at $y=0$ (neutral axis)

- q is shear flow:

$$q = \frac{V_{\text{longitudinal}}}{\Delta x} = \frac{V_T Q}{I}$$
Shearing Stresses

\[ f_v = \frac{V}{\Delta A} = \frac{V}{b \cdot \Delta x} \]

\[ f_{v\text{--ave}} = \frac{VQ}{Ib} \]

- \( f_v = 0 \) on the top/bottom
- \( b \text{ min may not be with } Q \text{ max} \)
- \( \text{with } h/4 \geq b, f_{v\text{-max}} \leq 1.008 f_{v\text{-ave}} \)
Rectangular Sections

\[ I = \frac{bh^3}{12} \quad Q = A\bar{y} = \frac{bh^2}{8} \]

\[ f_v = \frac{VQ}{Ib} = \frac{3V}{2A} \]

- \( f_{v\text{-max}} \) occurs at n.a.
Steel Beam Webs

- **W and S sections**
  - $b$ varies
  - stress in flange negligible
  - presume constant stress in web

\[
 f_{v\text{-max}} = \frac{3V}{2A} \approx \frac{V}{A_{\text{web}}}
\]
Shear Flow

- loads applied in plane of symmetry
- cut made perpendicular

\[ q = \frac{VQ}{I} \]
Shear Flow Quantity

- sketch from Q

\[ q = \frac{VQ}{I} \]
Connectors Resisting Shear

- plates with
  - nails
  - rivets
  - bolts
- splices

\[
\frac{V_{\text{longitudinal}}}{p} = \frac{VQ}{I}
\]

\[
nF_{\text{connector}} \geq \frac{VQ_{\text{connected area}}}{I} \cdot p
\]
Vertical Connectors

- *isolate an area with vertical interfaces*

\[ nF_{\text{connector}} \geq \frac{VQ_{\text{connected area}}}{I} \cdot p \]
Unsymmetrical Shear or Section

• member can bend and twist
  – not symmetric
  – shear not in that plane

• shear center
  – moments balance