Rigid Frames

- **rigid frames have no pins**
- **frame is all one body**
- **joints transfer moments and shear**
- **typically statically indeterminate**
- **types**
  - portal
  - gable
Rigid Frames

- behavior

(a) P#F rigid frame

(b) support here
legs spread, bending stress in beam only; none in legs

(c) force legs inward; legs now in bending; beam sags less

(d) fixed joint at bottom of legs; beam sags even less
Rigid Frames

- moments get redistributed
- deflections are smaller
- effective column lengths are shorter
- very sensitive to settling
Moment Redistribution

- **continuous slabs & beams with uniform loading**
  - joints similar to fixed ends, but can rotate
- **change in moment to center** $= \frac{wL^2}{8}$
  - $M_{\text{max}}$ for simply supported beam
Rigid Frames

- resists lateral loadings
- shape depends on stiffness of beams and columns
- $90^\circ$ maintained
Rigid Frames

- staggered truss
  - rigidity
  - clear stories
Rigid Frames

- connections
  - steel
  - concrete

http://nisee.berkeley.edu/godden
Braced Frames

- *pin connections*
- *bracing to prevent lateral movements*
Braced Frames

- types of bracing
  - knee-bracing
  - diagonal
  - X
  - K or chevron
  - shear walls
Shear Walls

• resist lateral load in plane with wall
Compression Members

- designed for strength & stresses
- designed for serviceability & deflection
- need to design for stability
  - ability to support a specified load without sudden or unacceptable deformations
Column Buckling

- axially loaded columns
- long & slender
  - unstable equilibrium = buckling
  - sudden and not good
Modeling

- can be modeled with a spring at mid-height
- when moment from deflection exceeds the spring capacity ... “boing”
- critical load $P$
Effect of Length

- long & slender
- short & stubby
Buckling Load

- related to deflected shape \((P\Delta)\)
- shape of sine wave
- Euler’s Formula
- smallest \(I\) governs

\[ P_{\text{critical}} = \frac{\pi^2 EI}{(L)^2} \]
Critical Stress

- short columns

\[ f_{\text{critical}} = \frac{P_{\text{actual}}}{A} < F_a \]

- slenderness ratio = \( \frac{L_e}{r} \) (L/d)

- radius of gyration = \( r = \sqrt{\frac{I}{A}} \)

\[
 f_{\text{critical}} = \frac{P_{\text{critical}}}{A} = \frac{\pi^2 EA r^2}{A (L_e)^2} = \frac{\pi^2 E}{\left(\frac{L_e}{r}\right)^2} \]

\[
 P_{\text{critical}} = \frac{\pi^2 EA}{\left(\frac{L_e}{r}\right)^2}
\]
Critical Stresses

- When a column gets stubby, $F_y$ will limit the load.
- Real world has loads with eccentricity.
- $C_c$ for steel and allowable stress.

\[
\frac{L_e}{r} > C_c = \sqrt{\frac{2\pi^2 E}{F_y}}
\]
Effective Length

- end conditions affect shape
- effective length factor, $K$ \[ L_e = K \cdot L \]
Bracing

- **bracing affects shape of buckle in one direction**
- **both should be checked!**
Centric & Eccentric Loading

• centric
  – allowable stress from strength or buckling

• eccentric
  – combined stresses
Combined Stresses

- axial + bending

\[ f_{\text{max}} = \frac{P}{A} + \frac{Mc}{I} \]

\[ M = P \cdot e \]

- design

\[ f_{\text{max}} \leq F_{cr} = \frac{f_{cr}}{F.S.} \]
Stress Limit Conditions

– ASD interaction formula

\[ \frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0 \]

– with biaxial bending

\[ \frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \]

interaction diagram
Stress Limit Conditions

- in reality, as the column flexes, the moment increases

- $P-\Delta$ effect

$$\frac{f_a}{F_a} + \frac{f_b \times (Magnification \ factor)}{F_{bx}} \leq 1.0$$
Rigid Frame Analysis

- **members see**
  - shear
  - axial force
  - bending

- **V & M diagrams**
  - plot on “outside”
Rigid Frame Analysis

- need support reactions
- free body diagram each member
- end reactions are equal and opposite on next member
- “turn” member like beam
- draw V & M
Rigid Frame Analysis

– FBD & M

• opposite end reactions at joints

\[ M_{BA} = \frac{P h}{2} \]
\[ M_{BC} = \frac{P h}{2} \]
\[ M_{CB} = \frac{P h}{2} \]
\[ M_{CD} = \frac{P h}{2} \]
Rigid Frame Design

- loads and combinations
  - usually uniformly distributed gravity loads
  - worst case for largest moments...
  - wind direction can increase moments
Rigid Frame Design

- frames & floors
  - rigid frame can have slab floors or slab with connecting beams
- other
  - slabs or plates on columns
Rigid Frame Design

- **floors – plates & slabs**
  - one-way behavior
    - side ratio > 1.5
    - “strip” beam
  - two-way behavior
    - more complex
Rigid Frame Design

- **columns in frames**
  - ends can be “flexible”
  - stiffness affected by beams and column = $EI/L$

\[
G = \Psi = \frac{\sum EI}{\sum EI} \frac{l_c}{l_b}
\]

- for the joint
  - $l_c$ is the column length of each column
  - $l_b$ is the beam length of each beam
  - measured center to center
Rigid Frame Design

- column effective length, \( k \)
Tools – Multiframe

• *in computer lab*
Tools – Multiframe

- **frame window**
  - define frame members
    - or pre-defined frame
  - select points, assign supports
  - select members, assign section
  - load window
  - select point or member, add point or distributed loads
**Tools – Multiframe**

- **to run analysis choose**
  - Analyze menu
    - Linear
- **plot**
  - choose options
- **results**
  - choose options