concrete construction: shear & deflection
Shear in Concrete Beams

- flexure combines with shear to form diagonal cracks
- horizontal reinforcement doesn’t help
- stirrups = vertical reinforcement
ACI Shear Values

- $V_u$ is at distance $d$ from face of support
- shear capacity: $V_c = \nu_c \times b_w d$

- where $b_w$ means thickness of web at n.a.
ACI Shear Values

- shear stress (beams)
  \[ \nu_c = 2\sqrt{f_c'} \]
  \[ \phi V_c = \phi 2\sqrt{f_c'} b_w d \]
  \( \phi = 0.75 \) for shear
  \( f_c' \) is in psi

- shear strength:
  \[ V_u \leq \phi V_c + \phi V_s \]
  - \( V_s \) is strength from stirrup reinforcement

\[ T = A_s f_t \]

Figure 13.17 Consideration for spacing of a single stirrup.
Stirrup Reinforcement

- **shear capacity:**

\[ V_s = \frac{A_v f_y d}{s} \]

- \( A_v \) = area in all legs of stirrups
- \( s \) = spacing of stirrup

- may need stirrups when concrete has enough strength!
### Required Stirrup Reinforcement

- **Spacing limits**

#### Table 3-8 ACI Provisions for Shear Design*

<table>
<thead>
<tr>
<th>Required area of stirrups, $A_v$ **</th>
<th>$V_u \leq \frac{\phi V_c}{2}$</th>
<th>$\phi V_c \geq V_u &gt; \frac{\phi V_c}{2}$</th>
<th>$V_u &gt; \phi V_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required</td>
<td>none</td>
<td>$\frac{50b_w s}{f_y}$</td>
<td>$\frac{(V_u - \phi V_c)s}{\phi f_y d}$</td>
</tr>
<tr>
<td>Recommended Minimum†</td>
<td></td>
<td>$\frac{A_v f_y}{50b_w}$</td>
<td>$\frac{\phi A_v f_y d}{V_u - \phi V_c}$</td>
</tr>
<tr>
<td>Stirrup spacing, $s$</td>
<td></td>
<td>$\frac{d}{2}$ or 24 in.</td>
<td>$\frac{d}{4}$ or 24 in. for $\left(V_u - \phi V_c\right) \leq \phi 4\sqrt{\phi \tau_c b_w d}$</td>
</tr>
<tr>
<td>Maximum†† (ACI 11.5.4)</td>
<td></td>
<td>$\frac{d}{2}$ or 24 in.</td>
<td>$\frac{d}{4}$ or 12 in. for $\left(V_u - \phi V_c\right) &gt; \phi 4\sqrt{\phi \tau_c b_w d}$</td>
</tr>
</tbody>
</table>

*Members subjected to shear and flexure only; $\phi V_c = \phi 2 \sqrt{\phi \tau_c b_w d}$, $\phi = 0.75$ (ACI 11.3.1.1)

**$A_v = 2 \times A_b$ for U stirrups; $f_y \leq 60$ ksi (ACI 11.5.2)

†A practical limit for minimum spacing is $d/4$

††Maximum spacing based on minimum shear reinforcement ($= A_v f_y/50b_w$) must also be considered (ACI 11.5.5.3).
Torsional Stress & Strain

- can see torsional stresses & twisting of axi-symmetrical cross sections
  - torque
  - remain plane
  - undistorted
  - rotates
- not true for square sections....
Shear Stress Distribution

- depend on the deformation
- \( \phi = \text{angle of twist} \)
  - measure
- can prove planar section doesn’t distort
Shearing Strain

- related to $\phi$

$$\gamma = \frac{\rho \phi}{L}$$

- $\rho$ is the radial distance from the centroid to the point under strain

- shear strain varies linearly along the radius: $\gamma_{\text{max}}$ is at outer diameter
Torsional Stress - Strain

- know $f_v = \tau = G \cdot \gamma$ and $\gamma = \frac{\rho \phi}{L}$
- so $\tau = G \cdot \frac{\rho \phi}{L}$
- where $G$ is the Shear Modulus
Torsional Stress - Strain

- from \( T = \Sigma \tau(\rho) \Delta A \)

- can derive \( T = \frac{\tau J}{\rho} \)

- where \( J \) is the polar moment of inertia

- elastic range \( \tau = \frac{T \rho}{J} \)
Shear Stress

- $\tau_{\text{max}}$ happens at outer diameter

- combined shear and axial stresses
  - maximum shear stress at 45° “twisted” plane
Shear Strain

- knowing \( \tau = G \cdot \frac{\rho \phi}{L} \) and \( \tau = \frac{T \rho}{J} \)

- solve: \( \phi = \frac{TL}{JG} \)

- composite shafts: \( \phi = \sum_i \frac{T_i L_i}{J_i G_i} \)
Noncircular Shapes

- torsion depends on J
- plane sections don’t remain plane
- $\tau_{\text{max}}$ is still at outer diameter

$$\tau_{\text{max}} = \frac{T}{c_1ab^2} \quad \phi = \frac{TL}{c_2ab^3G}$$

- where $a$ is longer side ($> b$)

<table>
<thead>
<tr>
<th>$a/b$</th>
<th>$c_1$</th>
<th>$c_2$</th>
</tr>
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<tbody>
<tr>
<td>1.0</td>
<td>0.208</td>
<td>0.1406</td>
</tr>
<tr>
<td>1.2</td>
<td>0.219</td>
<td>0.1661</td>
</tr>
<tr>
<td>1.5</td>
<td>0.231</td>
<td>0.1958</td>
</tr>
<tr>
<td>2.0</td>
<td>0.246</td>
<td>0.229</td>
</tr>
<tr>
<td>2.5</td>
<td>0.258</td>
<td>0.249</td>
</tr>
<tr>
<td>3.0</td>
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<td>0.263</td>
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<tr>
<td>4.0</td>
<td>0.282</td>
<td>0.281</td>
</tr>
<tr>
<td>5.0</td>
<td>0.291</td>
<td>0.291</td>
</tr>
<tr>
<td>10.0</td>
<td>0.312</td>
<td>0.312</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.333</td>
<td>0.333</td>
</tr>
</tbody>
</table>
Open Thin-Walled Sections

- with very large $a/b$ ratios:

$$\tau_{\text{max}} = \frac{T}{\frac{1}{3}ab^2}$$

$$\phi = \frac{TL}{\frac{1}{3}ab^3G}$$
Shear Flow in Closed Sections

- \( q \) is the internal shear force/unit length

\[
\tau = \frac{T}{2ta}
\]

\[
\phi = \frac{TL}{4ta^2} \sum_i \frac{s_i}{t_i}
\]

- \( a \) is the area bounded by the centerline
- \( s_i \) is the length segment, \( t_i \) is the thickness
Shear Flow in Open Sections

- each segment has proportion of $T$ with respect to torsional rigidity,

$$\tau_{\text{max}} = \frac{T t_{\text{max}}}{\frac{1}{3} \Sigma b_i t_i^3}$$

- total angle of twist:

$$\phi = \frac{TL}{\frac{1}{3} G \Sigma b_i t_i^3}$$

- I beams - web is thicker, so $\tau_{\text{max}}$ is in web
Torsional Shear Stress

- **twisting moment**
- **and beam shear**

![Diagram](image_url)

- *Figure R11.6.3.1—Addition of torsional and shear stresses*
Torsional Shear Reinforcement

- closed stirrups
- more longitudinal reinforcement
- area enclosed by shear flow
Development Lengths

- **required to allow steel to yield** \( (f_y) \)
- **standard hooks**
  - moment at beam end

- **splices**
  - lapped
  - mechanical connectors
Development Lengths

- $l_d$, embedment required both sides
- proper cover, spacing:
  - No. 6 or smaller
    \[ l_d = \frac{d_b F_y}{25 \sqrt{f_c'}} \quad \text{or 12 in. minimum} \]
  - No. 7 or larger
    \[ l_d = \frac{d_b F_y}{20 \sqrt{f_c'}} \quad \text{or 12 in. minimum} \]
Development Lengths

- **hooks**
  - **bend and extension**

  ![Figure 9-17](image1.png)
  **Figure 9-17**: Minimum requirements for 90° bar hooks.

  ![Figure 9-18](image2.png)
  **Figure 9-18**: Minimum requirements for 180° bar hooks.

- **minimum**

\[
 l_{dh} = \frac{1200d_b}{\sqrt{f'_c}}
\]
Development Lengths

• bars in compression

\[ l_d = \frac{0.02d_b F_y}{\sqrt{f'_c}} \leq 0.0003d_b F_y \]

• splices
  – tension minimum is function of \( l_d \) and splice classification
  – compression minimum
  – is function of \( d_b \) and \( F_y \)
Concrete Deflections

- elastic range
  - I transformed
  - $E_c$ (with $f'_c$ in psi)
    - normal weight concrete ($\sim 145 \text{ lb/ft}^3$)
      $E_c = 57,000 \sqrt{f'_c}$
    - concrete between 90 and 160 lb/ft$^3$
      $E_c = w_c^{1.5} 33 \sqrt{f'_c}$
  - cracked
    - I cracked
    - $E$ adjusted
Deflection Limits

- relate to whether or not beam supports or is attached to a damageable non-structural element
- need to check service live load and long term deflection against these

<table>
<thead>
<tr>
<th>Limit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>L/180</td>
<td>roof systems (typical) – live</td>
</tr>
<tr>
<td>L/240</td>
<td>floor systems (typical) – live + long term</td>
</tr>
<tr>
<td>L/360</td>
<td>supporting plaster – live</td>
</tr>
<tr>
<td>L/480</td>
<td>supporting masonry – live + long term</td>
</tr>
</tbody>
</table>