lecture twenty four

concrete construction: shear & deflection
Shear in Concrete Beams

- *flexure combines with shear to form diagonal cracks*
- *horizontal reinforcement doesn’t help*
- *stirrups = vertical reinforcement*
ACI Shear Values

- $V_u$ is at distance $d$ from face of support

- shear capacity: $V_c = \nu_c \times b_w d$

- where $b_w$ means thickness of web at n.a.
ACI Shear Values

- **shear stress** (beams)
  \[
  \nu_c = 2\sqrt{f'_c}
  \]
  \[
  \phi V_c = \phi 2\sqrt{f'_c} b_w d
  \]
  \[
  \phi = 0.75 \text{ for shear } f'_c \text{ is in psi}
  \]

- **shear strength:**
  \[
  V_u \leq \phi V_c + \phi V_s
  \]
  - \( V_s \) is strength from stirrup reinforcement

\[\text{Figure 13.17 Consideration for spacing of a single stirrup.}\]
Stirrup Reinforcement

• shear capacity:

\[ V_s = \frac{A_v f_y d}{s} \]

– \( A_v \) = area in all legs of stirrups
– \( s \) = spacing of stirrup

• may need stirrups when concrete has enough strength!
### Required Stirrup Reinforcement

**spacing limits**

#### Table 3-8 ACI Provisions for Shear Design*

<table>
<thead>
<tr>
<th></th>
<th>$V_u \leq \frac{\phi V_c}{2}$</th>
<th>$\phi V_c \geq V_u &gt; \frac{\phi V_c}{2}$</th>
<th>$V_u &gt; \phi V_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required area of stirrups, $A_v^{**}$</td>
<td>none</td>
<td>$\frac{50b_w s}{f_y}$</td>
<td>$(V_u - \phi V_c)s$</td>
</tr>
<tr>
<td>Stirrup spacing, $s$</td>
<td>Required</td>
<td>$\frac{A_v f_y}{50 b_w}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Recommended Minimum†</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Maximum‡‡ (ACI 11.5.4)</td>
<td>$\frac{d}{2}$ or 24 in.</td>
<td>$\frac{d}{2}$ or 24 in. for $(V_u - \phi V_c) \leq \phi 4 \sqrt{f'_c b_w d}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\frac{d}{4}$ or 12 in. for $(V_u - \phi V_c) &gt; \phi 4 \sqrt{f'_c b_w d}$</td>
</tr>
</tbody>
</table>

*Members subjected to shear and flexure only; $\phi V_c = \phi 2 \sqrt{f'_c b_w d}$, $\phi = 0.75$ (ACI 11.3.1.1)

**$A_v = 2 \times A_y$ for U stirrups; $f_y \leq 60$ ksi (ACI 11.5.2)

†A practical limit for minimum spacing is $d/4$

‡‡Maximum spacing based on minimum shear reinforcement (= $A_v f_y/50 b_w$) must also be considered (ACI 11.5.5.3).
Torsional Stress & Strain

• can see torsional stresses & twisting of axi-symmetrical cross sections
  – torque
  – remain plane
  – undistorted
  – rotates

• not true for square sections....
Shear Stress Distribution

- depend on the deformation
- $\phi = \text{angle of twist}$
  - measure
- can prove planar section doesn’t distort
Shearing Strain

• related to $\phi$

$$\gamma = \frac{\rho \phi}{L}$$

• $\rho$ is the radial distance from the centroid to the point under strain

• shear strain varies linearly along the radius: $\gamma_{max}$ is at outer diameter
Torsional Stress - Strain

• know \( f_v = \tau = G \cdot \gamma \) and \( \gamma = \frac{\rho \phi}{L} \)

• so \( \tau = G \cdot \frac{\rho \phi}{L} \)

• where \( G \) is the Shear Modulus
Torsional Stress - Strain

- from
  \[ T = \Sigma \tau(\rho) \Delta A \]
- can derive
  \[ T = \frac{\tau J}{\rho} \]
  - where \( J \) is the polar moment of inertia
  - elastic range
    \[ \tau = \frac{T \rho}{J} \]
Shear Stress

- $\tau_{\text{max}}$ happens at outer diameter

- combined shear and axial stresses
  - maximum shear stress at $45^\circ$ “twisted” plane
Shear Strain

- knowing \( \tau = G \cdot \frac{\rho \phi}{L} \) and \( \tau = \frac{T \rho}{J} \)

- solve: \( \phi = \frac{TL}{JG} \)

- composite shafts: \( \phi = \sum_i \frac{T_i L_i}{J_i G_i} \)
Noncircular Shapes

• torsion depends on $J$
• plane sections don’t remain plane
• $\tau_{\text{max}}$ is still at outer diameter

$$\tau_{\text{max}} = \frac{T}{c_1 ab^2} \quad \phi = \frac{TL}{c_2 ab^3 G}$$

- where $a$ is longer side ($>b$)

<table>
<thead>
<tr>
<th>$a/b$</th>
<th>$c_1$</th>
<th>$c_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.208</td>
<td>0.1406</td>
</tr>
<tr>
<td>1.2</td>
<td>0.219</td>
<td>0.1661</td>
</tr>
<tr>
<td>1.5</td>
<td>0.231</td>
<td>0.1958</td>
</tr>
<tr>
<td>2.0</td>
<td>0.246</td>
<td>0.229</td>
</tr>
<tr>
<td>2.5</td>
<td>0.258</td>
<td>0.249</td>
</tr>
<tr>
<td>3.0</td>
<td>0.267</td>
<td>0.263</td>
</tr>
<tr>
<td>4.0</td>
<td>0.282</td>
<td>0.281</td>
</tr>
<tr>
<td>5.0</td>
<td>0.291</td>
<td>0.291</td>
</tr>
<tr>
<td>10.0</td>
<td>0.312</td>
<td>0.312</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.333</td>
<td>0.333</td>
</tr>
</tbody>
</table>
Open Thin-Walled Sections

• with very large $a/b$ ratios:

$$\tau_{\text{max}} = \frac{T}{\frac{1}{3}ab^2}$$

$$\phi = \frac{TL}{\frac{1}{3}ab^3G}$$
Shear Flow in Closed Sections

- \( q \) is the internal shear force/unit length

\[
\tau = \frac{T}{2tA}
\]

\[
\phi = \frac{T_L}{4tA^2} \sum_i s_i t_i
\]

- \( A \) is the area bounded by the centerline
- \( s_i \) is the length segment, \( t_i \) is the thickness
Shear Flow in Open Sections

- each segment has proportion of $T$ with respect to torsional rigidity,

$$\tau_{\text{max}} = \frac{TT_{\text{max}}}{\frac{1}{3} \Sigma b_i t_i^3}$$

- total angle of twist:

$$\phi = \frac{TL}{\frac{1}{3} G \Sigma b_i t_i^3}$$

- I beams - web is thicker, so $\tau_{\text{max}}$ is in web
Torsional Shear Stress

- twisting moment
- and beam shear

Design torque may not be reduced because moment redistribution is not possible.

Fig. R11.6.3.1—Addition of torsional and shear stresses
Torsional Shear Reinforcement

- closed stirrups
- more longitudinal reinforcement
- area enclosed by shear flow

Fig. R11.6.3.6(a)—Space truss analogy

Fig. R11.6.3.6(b)—Definition of $A_{oh}$
Development Lengths

- required to allow steel to yield \( (f_y) \)
- standard hooks
  - moment at beam end

- splices
  - lapped
  - mechanical connectors
Development Lengths

- $l_d$, embedment required both sides
- proper cover, spacing:
  - No. 6 or smaller
    \[ l_d = \frac{d_b F_y}{25 \sqrt{f_c'}} \quad \text{or 12 in. minimum} \]
  - No. 7 or larger
    \[ l_d = \frac{d_b F_y}{20 \sqrt{f_c'}} \quad \text{or 12 in. minimum} \]
Development Lengths

- **hooks**
  - *bend and extension*

- **minimum**

\[ l_{dh} = \frac{1200d_b}{\sqrt{f_c'}} \]
Development Lengths

- **bars in compression**
  \[ l_d = \frac{0.02d_b F_y}{\sqrt{f'_c}} \leq 0.0003d_b F_y \]

- **splices**
  - tension minimum is function of \( l_d \) and splice classification
  - compression minimum
  - is function of \( d_b \) and \( F_y \)
Concrete Deflections

• elastic range
  – I transformed
  – $E_c$ (with $f'_c$ in psi)
    • normal weight concrete (~ 145 lb/ft$^3$)
      $$E_c = 57,000 \sqrt[3]{f'_c}$$
    • concrete between 90 and 160 lb/ft$^3$
      $$E_c = w_c^{1.5} 33 \sqrt[3]{f'_c}$$

• cracked
  – I cracked
  – $E$ adjusted
Deflection Limits

• relate to whether or not beam supports or is attached to a damageable non-structural element

• need to check service live load and long term deflection against these

| L/180  | roof systems (typical) – live |
| L/240  | floor systems (typical) – live + long term |
| L/360  | supporting plaster – live |
| L/480  | supporting masonry – live + long term |