Point Equilibrium 1
Lecture 4
Architectural Structures
ARCH 331
Point Equilibrium and Planar Trusses
Equilibrium

- balanced
- steady
- resultant of forces on a particle is 0
Equilibrium on a Point

- **analytically**

\[ R_x = \sum F_x = 0 \]
\[ R_y = \sum F_y = 0 \]

- *Newton convinces us it will stay at rest*
Equilibrium on a Point

- **Collinear force system**
  - ex: cables

\[ \sum F_{in-line} = 0 \]

\[
\begin{align*}
R_x &= \sum F_x = 0 \\
R_y &= \sum F_y = 0
\end{align*}
\]
Equilibrium on a Point

- concurrent force system
  - ex: cables

\[
R_x = \sum F_x = 0
\]

\[
R_y = \sum F_y = 0
\]
Free Body Diagram

- **FBD (sketch)**
- tool to see all forces on a body or a point including
  - external forces
  - weights
  - force reactions
  - internal forces
Free Body Diagram

- **determine point**
- **FREE it from:**
  - ground
  - supports & connections
- **draw all external forces acting ON the body**
  - reactions (supporting forces)
  - applied forces
  - gravity

Sign suspended from a strut and cable.

FBD of concurrent point B.

\[ F = m \cdot g \]
Free Body Diagram

• sketch FBD with relevant geometry
• resolve each force into components
  – known & unknown angles – name them
  – known & unknown forces – name them
• are any forces related to other forces?
• for the unknowns
• write only as many equilibrium equations as needed
• solve up to 2 equations
Free Body Diagram

• solve equations
  – most times 1 unknown easily solved
  – plug into other equation(s)

• common to have unknowns of
  – force magnitudes
  – force angles
Truss Structures

- ancient (?) wood
  - Romans 500 B.C.
- Renaissance revival
- 1800’s analysis
- efficient
Truss Structures
– analogous to cables and struts

(a) STABLE: pinned supports resist thrust

(b) UNSTABLE: substitution of roller support eliminates thrust resistance

(c) STABLE: wood strut resists thrust internally to form simple truss
Truss Structures

- comprised of straight members
- geometry with triangles is stable
- loads applied only at pin joints
Truss Structures

- 2 force members
  - forces in line, equal and opposite
  - compression
  - tension

- 3 members connected by 3 joints

- 2 more members need 1 more joint \[ b = 2n - 3 \]
Truss Structures

• compression and tension
**Truss Structures**

- **statically determinate**
- **indeterminate**
- **unstable**

![Truss Structures Diagram](image)

\[ b = 21 \quad n = 12 \quad 2(n) - 3 = 2(12) - 3 = 21 \]

\((a)\) Determinate.

\[ b = 16 \quad n = 10 \quad b = 16 < 2(10) - 3 = 17 \]

\((c)\) Unstable. 

\[ \text{Too few members—square panel is unstable} \]

\[ b = 18 \quad n = 10 \quad b = 18 > 2(10) - 3 = 17 \]

\((b)\) Indeterminate.

\[ \text{(Too many members)} \]
Trusses

• common designs
Trusses

• common designs
Trusses

• *uses*
  – roofs & canopies
  – long spans
  – lateral bracing
Truss Connections

• “pins”
Sainsbury Center, Foster 1978
two pin-connection supports
(typical of all trusses)

see detail

third pin connection at end trusses only
(makes truss and supporting columns
behave as a rigid frame
to minimize movement around end glazing)

tubular steel prism columns are cantilevered from foundation
(rigid base connection)

prism (3-sided) roof trusses
tubular cross-bracing between columns
Truss Analysis

- visualize compression and tension from deformed shape

http://nisee.berkeley.edu/godden
Truss Analysis

- **Method of Joints**
- **Graphical Methods**
- **Method of Sections**

- all rely on equilibrium
  - of bodies
  - internal equilibrium
Method of Joints

- isolate each joint
- enforce equilibrium in $F_x$ and $F_y$
- can find all forces

- long
- easy to mess up
Joint Cases

- two bodies connected

A

\[\begin{align*}
\text{equal} & \quad \text{or} \\
\text{equal and 0}
\end{align*}\]
Joint Cases

- three bodies with two in line
Joint Cases

- crossed
Tools – Multiframe

• in computer lab
Tools – Multiframe

- **frame window**
  - define truss members
    - or pre-defined truss
  - select points, assign supports
  - select members, assign *section* & assign *pin ends*

- **load window**
  - select points, add point load
Tools – Multiframe

- to run analysis choose
  - Analyze menu
    - Linear
- plot
  - choose options
- results
  - choose options