beams –
internal forces & diagrams
Beams

• span horizontally
  – floors
  – bridges
  – roofs

• loaded transversely by gravity loads

• may have internal axial force

• will have internal shear force

• will have internal moment (bending)
Beams

• transverse loading

• sees:
  – bending
  – shear
  – deflection
  – torsion
  – bearing

• behavior depends on cross section shape
Beams

- bending
  - bowing of beam with loads
  - one edge surface stretches
  - other edge surface squishes
Beam Stresses

- stress = relative force over an area
  - tensile
  - compressive
  - bending

- tension and compression + ...
Beam Stresses

unreinforced concrete beam fails in tension (cracks on bottom)

steel reinforcing in bottom of beam resists tension
Beam Stresses

- tension and compression
  - causes moments

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**Beam Stresses**

- *prestress or post-tensioning*
  - put stresses in tension area to “pre-compress”
Beam Stresses

• shear – horizontal & vertical
Beam Stresses

• shear – horizontal & vertical
Beam Stresses

- **shear – horizontal**
Beam Deflections

- depends on
  - load
  - section
  - material
Beam Deflections

• “moment of inertia”
Beam Styles

- **vierendeel**
- **open web joists**
- **manufactured**
Internal Forces

• **trusses**
  – axial only, (compression & tension)

\[ F \rightarrow A \quad \rightarrow B \quad \rightarrow F \]
\[ F \leftarrow A \quad \rightarrow F' \quad \leftarrow F' \quad \rightarrow B \quad \rightarrow F \]

• **in general**
  – axial force
  – shear force, \( V \)
  – bending moment, \( M \)
Beam Loading

- concentrated force
- concentrated moment
  - spandrel beams
Beam Loading

- uniformly distributed load (line load)
- non-uniformly distributed load
  - hydrostatic pressure = $\gamma h$
  - wind loads

(c) Nonuniformly distributed load.
Beam Supports

- **statically determinate**

  - simply supported (most common)
  - overhang
  - cantilever

- **statically indeterminate**

  - continuous (most common case when $L_1 = L_2$)
  - Propped
  - Restrained
Beam Supports

• in the real world, modeled type

(a) Beam supported by a neoprene pad.

(b) Timber beam–column connection with T-plate.
Internal Forces in Beams

• *like method of sections / joints*
  – no axial forces
• *section must be in equilibrium*
• *want to know where biggest internal forces and moments are for designing*
V & M Diagrams

- tool to locate $V_{\text{max}}$ and $M_{\text{max}}$ (at $V = 0$)
- necessary for designing
- have a different sign convention than external forces, moments, and reactions

![Diagram showing V & M Diagrams with (+)V and (+)M indicating the sign conventions]
Sign Convention

• shear force, V:
  – cut section to LEFT
  – if $\sum F_y$ is positive by statics, V acts down and is POSITIVE
  – beam has to resist shearing apart by V
Shear Sign Convention

(+) Shear.

(-) Shear.

(+) Shear.

(-) Shear.
Sign Convention

- **bending moment, M:**
  - cut section to LEFT
  - if $\sum M_{\text{cut}}$ is clockwise, M acts ccw and is **POSITIVE** – flexes into a “smiley” beam has to resist bending apart by M
Bending Moment Sign Convention

(+) Moment.

Holds Water

(+) Moment.

Negative Moment.

SHeds Water

(-) Moment.
Deflected Shape

- **positive bending moment**
  - tension in bottom, compression in top
- **negative bending moment**
  - tension in top, compression in bottom
- **zero bending moment**
  - inflection point
Constructing V & M Diagrams

- along the beam length, plot V, plot M

V

M

load diagram
Mathematical Method

- cut sections with $x$ as width
- write functions of $V(x)$ and $M(x)$
Method 1: Equilibrium

- cut sections at important places
- plot V & M

\[ V \]

\[ M \]

\[ L/2 \]
Method 1: Equilibrium

- **important places**
  - supports
  - concentrated loads
  - start and end of distributed loads
  - concentrated moments

- **free ends**
  - zero forces
Method 2: Semigraphical

- by knowing
  - area under loading curve = change in V
  - area under shear curve = change in M
  - concentrated forces cause “jump” in V
  - concentrated moments cause “jump” in M

\[
V_D - V_C = - \int_{x_C}^{x_D} w \, dx \\
M_D - M_C = \int_{x_C}^{x_D} V \, dx
\]
Method 2

- relationships
Method 2: Semigraphical

- $M_{\text{max}}$ occurs where $V = 0$ (calculus)
Curve Relationships

• integration of functions
• line with 0 slope, integrates to sloped

• ex: load to shear, shear to moment
Curve Relationships

- line with slope, integrates to parabola

- ex: load to shear, shear to moment
**Curve Relationships**

- **parabola, integrates to 3\textsuperscript{rd} order curve**

- **ex: load to shear, shear to moment**
Basic Procedure with Sections

1. Find reaction forces & moments
   Plot axes, underneath beam load diagram

2. Starting at left

3. Shear is 0 at free ends

4. Shear has 2 values at point loads

5. Sum vertical forces at each section
Basic Procedure with Sections

M:

6. Starting at left
7. Moment is 0 at free ends
8. Moment has 2 values at moments
9. **Sum moments at each section**
10. **Maximum moment is where shear = 0!** (locate where V = 0)
Basic Procedure by Curves

1. **Find reaction forces & moments**
   Plot axes, underneath beam load diagram

2. **Starting at left**
3. **Shear is 0 at free ends**
4. **Shear jumps with concentrated load**
5. **Shear changes with area under load**
Basic Procedure by Curves

M:

6. Starting at left
7. Moment is 0 at free ends
8. Moment jumps with moment
9. Moment changes with area under V
10. Maximum moment is where shear = 0! (locate where V = 0)
Shear Through Zero

- slope of $V$ is $w$ (-$w$:1)

$$x \cdot w = V_A \quad \Rightarrow \quad x = \frac{V_A}{w}$$
Parabolic Shapes

- cases

- \( \uparrow \text{ fast, then slow} \)
- \( \uparrow \text{ slow, then fast} \)
- \( \downarrow \text{ fast, then slow} \)
- \( \downarrow \text{ slow, then fast} \)
Deflected Shape & $M(x)$

- $M(x)$ gives shape indication
- boundary conditions must be met

![Diagram of deflected shape and bending moment](image)
Boundary Conditions

• at pins, rollers, fixed supports: \( y = 0 \)

• at fixed supports: \( \theta = 0 \)

• at inflection points from symmetry: \( \theta = 0 \)

• \( y_{\text{max}} \) at \( \frac{dy}{dx} = 0 \)
Tabulated Beam Formulas

- how to read charts

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD

Total Equiv. Uniform Load . . . . . = \(wl\)

\[ R = V \quad . . . . . . . . . . . = \frac{wl}{2} \]

\[ V_x \quad . . . . . . . . . . . = w \left( \frac{l}{2} - x \right) \]

\[ M \max. \ (at \ center) \quad . . . . . = \frac{wl^2}{8} \]

\[ M_x \quad . . . . . . . . . . . = \frac{wx}{2} (l - x) \]

\[ \Delta \max. \ (at \ center) \quad . . . . . = \frac{5 \, wl^4}{384 \, EI} \]

\[ \Delta x \quad . . . . . . . . . . . = \frac{wx}{24EI} \left( l^3 - 2lx^2 + x^3 \right) \]
Tools

- software & spreadsheets help
- [http://www.rekenwonder.com/atlas.htm](http://www.rekenwonder.com/atlas.htm)
Tools – Multiframe

• in computer lab
Tools – Multiframe

• frame window
  – define beam members
  – select points, assign supports
  – select members, assign section

• load window
  – select point or member, add point or distributed loads
Tools – Multiframe

- to run analysis choose
  - Analyze menu
    - Linear
- plot
  - choose options
  - double click (all)
- results
  - choose options