Rigid Frames

- **rigid frames have no pins**
- frame is all one body
- joints transfer moments and shear
- typically statically indeterminate
- types
  - portal
  - gable

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Rigid Frames

- behavior

- moments get redistributed
- deflections are smaller
- effective column lengths are shorter
- very sensitive to settling
**Moment Redistribution**

- continuous slabs & beams with uniform loading
  - joints similar to fixed ends, but can rotate
- change in moment to center = \( \frac{wL^2}{8} \)
  - \( M_{\text{max}} \) for simply supported beam

**Rigid Frames**

- resists lateral loadings
- shape depends on stiffness of beams and columns
- 90° maintained

**Rigid Frames**

- staggered truss
  - rigidity
  - clear stories

**Rigid Frames**

- connections
  - steel
  - concrete
Braced Frames

- pin connections
- bracing to prevent lateral movements

Shear Walls

- resist lateral load in plane with wall

Compression Members

- designed for strength & stresses
- designed for serviceability & deflection
- need to design for stability
  - ability to support a specified load without sudden or unacceptable deformations
Column Buckling

- axially loaded columns
- long & slender
  - unstable equilibrium = buckling
  - sudden and not good

Modeling

- can be modeled with a spring at mid-height
- when moment from deflection exceeds the spring capacity ...
  “boing”
- critical load $P$

Effect of Length

- long & slender
- short & stubby

Buckling Load

- related to deflected shape ($P\Delta$)
- shape of sine wave
- Euler’s Formula
- smallest $I$ governs

$$P_{\text{critical}} = \frac{\pi^2 EI}{(L)^2}$$
Critical Stress

- Short columns
  \[ f_{\text{critical}} = \frac{P_{\text{actual}}}{A} < F_a \]
- Slenderness ratio = \( L_e/r \) (L/d)
- Radius of gyration = \( r = \sqrt{\frac{I}{A}} \)

\[ f_{\text{critical}} = \frac{P_{\text{critical}}}{A} = \frac{\pi^2 EA r^2}{A(L_e/r)^2} = \frac{\pi^2 E}{(L_e/r)^2} \]

\[ P_{\text{critical}} = \frac{\pi^2 EA}{(L_e/r)^2} \]

Effective Length

- End conditions affect shape
- Effective length factor, \( K \)
  \[ L_e = K \cdot L \]

Critical Stresses

- When a column gets stubby, \( F_y \) will limit the load
- Real world has loads with eccentricity
- \( C_c \) for steel and allowable stress

\[ \frac{L_e}{r} > C_c = \sqrt{\frac{2\pi^2 E}{F_y}} \]

Bracing

- Bracing affects shape of buckle in one direction
- Both should be checked!
**Centric & Eccentric Loading**

- **centric**
  - allowable stress from strength or buckling
- **eccentric**
  - combined stresses

**Combined Stresses**

- **axial + bending**
  \[ f_{max} = \frac{P}{A} + \frac{Mc}{I} \]
  \[ M = P \cdot e \]

- **design**
  \[ f_{max} \leq F_{cr} = \frac{f_{cr}}{F.S.} \]

**Stress Limit Conditions**

- **ASD interaction formula**
  \[ \frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0 \]
- **with biaxial bending**
  \[ \frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \]

**Stress Limit Conditions**

- **in reality, as the column flexes, the moment increases**

  - **P-\Delta effect**
  \[ \frac{f_a}{F_a} + \frac{f_b \times \text{(Magnification factor)}}{F_{bx}} \leq 1.0 \]
Rigid Frame Analysis

• members see
  – shear
  – axial force
  – bending
• V & M diagrams
  – plot on “outside”

Rigid Frame Analysis

– need support reactions
– free body diagram each member
– end reactions are equal and opposite on next member
– “turn” member like beam
– draw V & M

Rigid Frame Analysis

– FBD & M
  • opposite end reactions at joints

Rigid Frame Design

• loads and combinations
  – usually uniformly distributed gravity loads
  – worst case for largest moments...
  – wind direction can increase moments
Rigid Frame Design

• frames & floors
  – rigid frame can have slab floors or slab with connecting beams

• other
  – slabs or plates on columns

Rigid Frame Design

• columns in frames
  – ends can be “flexible”
  – stiffness affected by beams and column = $EI/l$

\[
G = \Psi = \frac{\sum EI}{\sum l_c} = \frac{\sum l_c}{\sum l_b}
\]
  – for the joint
  • $l_c$ is the column length of each column
  • $l_b$ is the beam length of each beam
  • measured center to center

Rigid Frame Design

• floors – plates & slabs
  – one-way behavior
    • side ratio $> 1.5$
    • “strip” beam
  – two-way behavior
    • more complex
Tools – Multiframe

• in computer lab

• frame window
  – define frame members
  – select points, assign supports
  – select members, assign section
  – load window
  – select point or member, add point or distributed loads

Tools – Multiframe

• to run analysis choose
  – Analyze menu
    • Linear

• plot
  – choose options

• results
  – choose options