steel construction: columns & tension members

Design Methods (revisited)
- know
  - loads or lengths
- select
  - section or load
  - adequate for strength and no buckling

Structural Steel
- standard rolled shapes (W, C, L, T)
- tubing
- pipe
- built-up

Allowable Stress Design (ASD)
- AICS 9th ed
  \[ F_a = \frac{f_{\text{critical}}}{F.S.} = \frac{12\pi^2 E}{23(Kl/r)^2} \]
- slenderness ratio \( \frac{Kl}{r} \)
  - for \( kl/r \geq C_c \)
  \[ = 126.1 \text{ with } F_y = 36 \text{ ksi} \]
  \[ = 107.0 \text{ with } F_y = 50 \text{ ksi} \]
C_c and Euler’s Formula

- KL/r < C_c
  - short and stubby
  - parabolic transition

- KL/r > C_c
  - Euler’s relationship
  - < 200 preferred

\[ C_c = \sqrt{\frac{2\pi^2E}{F_y}} \]

Short / Intermediate

- L_e/r < C_c
  \[ F_a = \left[1 - \left(\frac{KL}{r}\right)^2\right] \frac{F_y}{F.S.} \]
  - where
  \[ F.S. = \frac{5}{3} + \frac{3(KL/r)}{8C_c} - \frac{(KL/r)^3}{8C_c^3} \]

Unified Design

- limit states for failure
  \[ P_a \leq \frac{P_n}{\Phi} \]
  \[ \phi_c = 0.90 \quad P_n = F_{cr} A_g \quad P_u \leq \phi_c \frac{P_n}{\Omega} \]

1. yielding \[ KL \leq 4.71 \sqrt{\frac{E}{F_y}} \quad \text{or} \quad F_e \geq 0.44F_y \]
2. buckling \[ KL > 4.71 \sqrt{\frac{E}{F_y}} \quad \text{or} \quad F_e < 0.44F_y \]

F_e – elastic buckling stress (Euler)
Unified Design

\[ P_n = F_{cr}A_g \]

- for \( \frac{KL}{r} \leq 4.71 \)
\[ F_{cr} = \frac{E}{F_y} \left( \frac{F_y}{F_e} \right)^{\frac{F_y}{F_e}} \]

- for \( \frac{KL}{r} > 4.71 \)
\[ F_{cr} = 0.877F \]

- where \( F_e = \frac{\pi^2 E}{(KL/r)^2} \)

Procedure for Analysis

1. calculate \( KL/r \)
   \- biggest of \( KL/r \) with respect to x axes and y axis

2. find \( F_a \) or \( F_{cr} \) from appropriate equation
   \- tables are available

3. compute \( P_{allowable} = F_a \cdot A \) or \( P_n = F_{cr}A_g \)
   \- or find \( f_{actual} = P/A \)

4. is \( P \leq P_{allowable} \) (or \( P_n \leq \phi P_n \))?
   \- yes: ok
   \- no: insufficient capacity and no good

Procedure for Design

1. guess a size (pick a section)

2. calculate \( KL/r \)
   \- biggest of \( KL/r \) with respect to x axes and y axis

3. find \( F_a \) or \( F_{cr} \) from appropriate equations
   \- or find a chart

4. compute \( P_{allowable} = F_a \cdot A \) (or \( P_n/\Omega = F_{cr}A_g \))
   \- or find \( f_{actual} = P/A \)

Procedure for Design (cont’d)

5. is \( P \leq P_{allowable} \) or \( P_n \leq \phi P_n \)?
   \- yes: ok
   \- no: pick a bigger section and go back to step 2.

6. check design efficiency
   \- percentage of stress = \( \frac{P}{P_c} \cdot 100\% \)
   \- if between 90-100%: good
   \- if < 90%: pick a smaller section and go back to step 2.
Column Charts, $F_a$ (pg. 361-364)

Table 10.1  Allowable stress for compression members ($F_y = 36$ ksi and $F_y = 250$ MPa).

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Column Charts, $\phi F_{cr}$

Available Critical Stress, $\phi F_{cr}$, for Compression Members, ksi ($F_y = 50$ ksi and $\phi = 0.90$)

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Beam-Column Design

• moment magnification ($P - \Delta$)

$$M_u = B_1 M_{\text{max}} - \text{factored} \quad B_1 = \frac{C_m}{1 - (P_u / P_{el})}$$

$C_m$ – modification factor for end conditions

$= 0.6 - 0.4(M_1/M_2)$ or

0.85 restrained, 1.00 unrestrained

$P_{el}$ – Euler buckling strength

$P_{el} = \frac{\pi^2 EA}{(Kl/r)^2}$
Beam-Column Design

- LRFD (Unified) Steel
  - for \( \frac{P_r}{P_c} \geq 0.2 \):
    \[
    \frac{P_u}{\phi_c P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) \leq 1.0
    \]
  - for \( \frac{P_r}{P_c} < 0.2 \):
    \[
    \frac{P_u}{2 \phi_c P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) \leq 1.0
    \]

\( P_r \) is required, \( P_c \) is capacity
\( \phi_c \) - resistance factor for compression = 0.9
\( \phi_b \) - resistance factor for bending = 0.9

Design Steps Knowing Loads (revisited)

1. assume limiting stress
   - buckling, axial stress, combined stress
2. solve for \( r, A \) or \( S \)
3. pick trial section
4. analyze stresses
5. section ok?
6. stop when section is ok

Rigid Frame Design (revisited)

- columns in frames
  - ends can be “flexible”
  - stiffness affected by beams and column = \( EI/L \)

\[
G = \Psi = \frac{\sum EI}{l_c} \quad \frac{\sum EI}{l_b}
\]

- for the joint
  - \( l_c \) is the column length of each column
  - \( l_b \) is the beam length of each beam
  - measured center to center

Rigid Frame Design (revisited)

- column effective length, \( k \)
**Tension Members**

- steel members can have holes
- reduced area
  \[ A_n = A_g - A_{of\ all\ holes} + t\sum_{4g} s \]
- increased stress

**Effective Net Area**

- likely path to “rip” across
- bolts divide transferred force too
- shear lag \[ A_e \leq A_n U \]

**Tension Members**

- limit states for failure
  \[ P_a \leq \frac{P_n}{\Omega} \quad P_u \leq \phi_t P_n \]
- yielding \[ \phi_t = 0.90 \quad P_n = F_y A_g \]
- rupture* \[ \phi_t = 0.75 \quad P_n = F_u A_e \]

\( A_g \) - gross area
\( A_e \) - effective net area
(holes 1/8" + d)
\( F_u \) = the tensile strength of the steel (ultimate)

\( P_u \) = the ultimate load capacity of the member
\( P_n \) = the nominal load capacity of the member
\(\Omega\) = a constant (usually 3.0)
\( P_a \) = the applied load
\( F_y \) = the yield stress of the steel
\( F_u \) = the ultimate tensile strength of the steel
\( t \) = thickness of the member
\( s \) = the distance between holes
\( A_{of\ all\ holes} \) = area of all holes

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(AISC - Steel Structures of the Everyday)