Structural Math

- quantify environmental loads
  - how big is it?
- evaluate geometry and angles
  - where is it?
  - what is the scale?
  - what is the size in a particular direction?
- quantify what happens in the structure
  - how big are the internal forces?
  - how big should the beam be?

Physics for Structures

- measures
  - US customary & SI

<table>
<thead>
<tr>
<th>Units</th>
<th>US</th>
<th>SI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>in, ft, mi</td>
<td>mm, cm, m</td>
</tr>
<tr>
<td>Volume</td>
<td>gallon</td>
<td>liter</td>
</tr>
<tr>
<td>Mass</td>
<td>lb mass</td>
<td>g, kg</td>
</tr>
<tr>
<td>Force</td>
<td>lb force</td>
<td>N, kN</td>
</tr>
<tr>
<td>Temperature</td>
<td>F</td>
<td>C</td>
</tr>
</tbody>
</table>
Physics for Structures

- **Scalars** – any quantity
- **Vectors** - quantities with direction
  - like displacements
  - summation results in the “straight line path” from start to end
  - normal vector is perpendicular to something

Language

- **Symbols for operations**: +, -, /, x
- **Symbols for relationships**: (), =, <, >
- **Algorithms**
  - cancellation
  - factors
  - signs
  - ratios and proportions
  - power of a number
  - conversions, ex. $1X = 10Y$
  - operations on both sides of equality

$$\frac{2 \times 5}{6} = \frac{2}{6} = \frac{2 \times 3}{3} = 1$$
$$\frac{x}{6} = \frac{1}{3}$$

$$10^3 = 1000$$

$$\frac{10Y}{1X} or \frac{1X}{10Y} = 1$$

On-line Practice

- **eCampus / Study Aids**

Geometry

- **Angles**
  - right $= 90^\circ$
  - acute $< 90^\circ$
  - obtuse $> 90^\circ$
  - $\pi = 180^\circ$

- **Triangles**
  - area $= \frac{b \times h}{2}$
  - hypotenuse
  - total of angles $= 180^\circ$

$$AB^2 + AC^2 = BC^2$$
Geometry

- lines and relation to angles
  - parallel lines can’t intersect
  - perpendicular lines cross at 90°
  - intersection of two lines is a point
  - opposite angles are equal when two lines cross

- sides of two angles are parallel and intersect opposite way, the angles are supplementary - the sum is 180°
- two angles that sum to 90° are said to be complimentary

Forces & Moments

- intersection of a line with parallel lines results in identical angles
- two lines intersect in the same way, the angles are identical
- intersection of a line with parallel lines results in identical angles
- two lines intersect in the same way, the angles are identical
**Geometry**

- similar triangles have proportional sides

\[
\frac{AB}{AD} = \frac{AC}{AE} = \frac{BC}{DE}
\]

\[
\frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'}
\]

**Trigonometry**

- for right triangles

\[
\sin \alpha = \frac{AB}{CB} = \frac{\text{opposite side}}{\text{hypotenuse}}
\]

\[
\cos \alpha = \frac{AC}{CB} = \frac{\text{adjacent side}}{\text{hypotenuse}}
\]

\[
\tan \alpha = \frac{AB}{AC} = \frac{\text{opposite side}}{\text{adjacent side}}
\]

**SOHCAHTOA**

**Trigonometry**

- cartesian coordinate system
  - origin at 0,0
  - coordinates in (x,y) pairs
  - x & y have signs

**Trigonometry**

- for angles starting at positive x
  - sin is y side
  - cos is x side

\sin<0 for 180-360°
\cos<0 for 90-270°
\tan<0 for 90-180°
\tan<0 for 270-360°
Trigonometry

• cartesian coordinate system
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Quadrant I
Quadrant II
Quadrant III
Quadrant IV

Trigonometry

• for angles starting at positive x
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Algebra

• equations (something = something)
• constants
  – real numbers or shown with a, b, c...
• unknown terms, variables
  – names like R, F, x, y
• linear equations
  – unknown terms have no exponents
• simultaneous equations
  – variable set satisfies all equations

Trigonometry

• for all triangles
  – sides A, B & C are opposite angles α, β & γ

– LAW of SINES
  \[
  \frac{\sin \alpha}{A} = \frac{\sin \beta}{B} = \frac{\sin \gamma}{C}
  \]

– LAW of COSINES
  \[
  A^2 = B^2 + C^2 - 2BC \cos \alpha
  \]
Algebra

• solving one equation
  – only works with one variable
  – ex: \(2x - 1 = 0\)
    • add to both sides \(2x - 1 + 1 = 0 + 1\)
    \[2x = 1\]
    • divide both sides \[\frac{2x}{2} = \frac{1}{2}\]
    \[x = \frac{1}{2}\]

• solving two equations
  – only works with two variables
  – ex: \(2x + 3y = 8\)
    • look for term similarity \(12x - 3y = 6\)
    • can we add or subtract to eliminate one term?
    • add \(2x + 3y + 12x - 3y = 8 + 6\)
      \[14x = 14\]
      • get x by itself on a side \[\frac{14x}{14} = \frac{14}{14} = x = 1\]

Algebra

• solving one equations
  – only works with one variable
  – ex: \(2x - 1 = 4x + 5\)
    • subtract from both sides \(2x - 1 - 2x = 4x + 5 - 2x\)
    \[2x - 1 - 2x = 4x + 5 - 2x\]
    • subtract from both sides \(-1 - 5 = 2x + 5 - 5\)
    \[-1 - 5 = 2x + 5 - 5\]
    • divide both sides \[\frac{-6}{2} = \frac{-3 \cdot 2}{2} = \frac{2x}{2}\]
    \[\frac{-6}{2} = \frac{-3 \cdot 2}{2} = \frac{2x}{2}\]
    • get x by itself on a side \[x = -3\]

Forces

• statics
  – physics of forces and reactions on bodies and systems
  – equilibrium (bodies at rest)

• forces
  – something that exerts on an object:
    • motion
    • tension
    • compression
Force

- “action of one body on another that affects the state of motion or rest of the body”
- Newton’s 3rd law:
  - for every force of action there is an equal and opposite reaction along the same line

Force Characteristics

- applied at a point
- magnitude
  - Imperial units: lb, k (kips)
  - SI units: N (newtons), kN
- direction

Forces on Rigid Bodies

- for statics, the bodies are ideally rigid
- can translate and rotate
- internal forces are
  - in bodies
  - between bodies (connections)
- external forces act on bodies

Transmissibility

- the force stays on the same line of action
- truck can’t tell the difference
- only valid for EXTERNAL forces
**Force System Types**

- **collinear**

  ![Collinear forces diagram](https://via.placeholder.com/150)

  Collinear—All forces acting along the same straight line.

- **coplanar**

  ![Coplanar forces diagram](https://via.placeholder.com/150)

  Coplanar—all forces are parallel and act in the same plane.

- **space**

  ![Space forces diagram](https://via.placeholder.com/150)

  Noncoplanar, concurrent—all forces act on a common point but do not all lie in the same plane.

  Noncoplanar, nonconcurrent—all forces act on different points.

**Adding Vectors**

- **graphically**

  - parallelogram law
    - diagonal
    - long for 3 or more vectors
  
  - tip-to-tail
    - more convenient with lots of vectors
**Force Components**

- convenient to resolve into 2 vectors
- at right angles
- in a “nice” coordinate system
- \( \theta \) is between \( F_x \) and \( F \) from \( F_x \)

\[
F_x = F \cos \theta \\
F_y = F \sin \theta \\
F = \sqrt{F_x^2 + F_y^2} \\
\tan \theta = \frac{F_y}{F_x}
\]

**Trigonometry**

- \( F_x \) is negative
  - 90\(^\circ\) to 270\(^\circ\)
- \( F_y \) is negative
  - 180\(^\circ\) to 360\(^\circ\)
- \( \tan \) is positive
  - quads I & III
- \( \tan \) is negative
  - quads II & IV

**Component Addition**

- find all x components
- find all y components
- find sum of x components, \( R_x \) (resultant)
- find sum of y components, \( R_y \)

\[
R = \sqrt{R_x^2 + R_y^2} \\
\tan \theta = \frac{R_y}{R_x}
\]

**Alternative Trig for Components**

- doesn’t relate angle to axis direction
- \( \phi \) is “small” angle between \( F \) and EITHER \( F_x \) or \( F_y \)
- no sign out of calculator!
- have to choose RIGHT trig function, resulting direction (sign) and component axis
Friction
• resistance to movement
• contact surfaces determine \( \mu \)
• proportion of normal force (\( \perp \))
  – opposite to slide direction
  – static > kinetic

\[ F = \mu N \]

Cables
• simple
• uses
  – suspension bridges
  – roof structures
  – transmission lines
  – guy wires, etc.
• have same tension all along
• can’t stand compression

Cables Structures
• use high-strength steel
• need
  – towers
  – anchors
• don’t want movement

Cable Structures

http://nisee.berkeley.edu/godden

http://nisee.berkeley.edu/godden
**Cable Loads**

- straight line between forces
- with one force
  - concurrent
  - symmetric

**Cable-Stayed Structures**

- diagonal cables support horizontal spans
- typically symmetrical
- Patcenter, Rogers 1986

**Patcenter, Rogers 1986**

- column free space
- roof suspended
- solid steel ties
- steel frame supports masts
Patcenter, Rogers 1986

- dashes – cables pulling

![Patcenter, load path diagram.](image)

Moments

- forces have the tendency to make a body rotate about an axis

- same translation but different rotation

Moments

- a force acting at a different point causes a different moment:
Moments

- defined by magnitude and direction
- units: N·m, k·ft
- direction: + ccw (right hand rule)  
  - cw
- value found from $F$ and $\perp$ distance

\[ M = F \cdot d \]

- $d$ also called “lever” or “moment” arm

Moments

- with same $F$:

\[ M_A = F \cdot d_1 < M_A = F \cdot d_2 \]

(bigger)

Moments

- additive with sign convention
- can still move the force along the line of action

Varignon’s Theorem

- resolve a force into components at a point and finding perpendicular distances
- calculate sum of moments
- equivalent to original moment

- makes life easier!
- geometry
- when component runs through point, $d=0$
Moments of a Force

- moments of a force
  - introduced in Physics as “Torque Acting on a Particle”
  - and used to satisfy rotational equilibrium

Physics and Moments of a Force

- my Physics book:

Moment Couples

- 2 forces
  - same size
  - opposite direction
  - distance d apart
  - cw or ccw
  
  \[ M = F \cdot d \]

- not dependant on point of application

  \[ M = F \cdot d_1 - F \cdot d_2 \]
Moment Couples

• added just like moments caused by one force
• can replace two couples with a single couple

Equivalent Force Systems

• two forces at a point is equivalent to the resultant at a point
• resultant is equivalent to two components at a point
• resultant of equal & opposite forces at a point is zero
• put equal & opposite forces at a point (sum to 0)
• transmission of a force along action line

Moment Couples

• moment couples in structures

Force-Moment Systems

• single force causing a moment can be replaced by the same force at a different point by providing the moment that force caused
• moments are shown as arched arrows
Force-Moment Systems

- A force-moment pair can be replaced by a force at another point causing the original moment.

Parallel Force Systems

- Forces are in the same direction.
- Can find resultant force.
- Need to find location for equivalent moments.