Mechanics of Materials

- external loads and their effect on deformable bodies
- use it to answer question if structure meets requirements of
  - stability and equilibrium
  - strength and stiffness
- other principle building requirements
  - economy, functionality and aesthetics

Knowledge Required

- material properties
- member cross sections
- ability of a material to resist breaking
- structural elements that resist excessive
  - deflection
  - deformation

Figure 2.34: An example of tension on a cantilever beam.
Problem Solving

1. STATICS:
   equilibrium of external forces, internal forces, stresses
2. GEOMETRY:
   cross section properties, deformations and conditions of geometric fit, strains
3. MATERIAL PROPERTIES:
   stress-strain relationship for each material obtained from testing

Stress

- stress is a term for the intensity of a force, like a pressure
- internal or applied
- force per unit area

\[ stress = f = \frac{P}{A} \]

Design

- materials have a critical stress value where they could break or yield
  - ultimate stress
  - yield stress
  - compressive stress
  - fatigue strength
  - (creep & temperature)

Design (cont)

- we’d like \( f_{\text{actual}} \ll F_{\text{allowable}} \)
- stress distribution may vary: average
- uniform distribution exists IF the member is loaded axially (concentric)
Scale Effect

- **model scale**
  - material weights by volume, small section areas
- **structural scale**
  - much more material weight, bigger section areas
- scale for strength is not proportional:
  \[
  \frac{\gamma L^3}{L^2} = \gamma L
  \]

Normal Stress (direct)

- normal stress is normal to the cross section
  - stressed area is perpendicular to the load
  \[
  f_{torc} = \frac{P}{A}
  \]

Shear Stress

- stress parallel to a surface
  \[
  f_v = \frac{P}{A} = \frac{P}{td}
  \]

Bearing Stress

- stress on a surface by contact in compression
  \[
  f_p = \frac{P}{A} = \frac{P}{td}
  \]

Figure 5.7 Two columns with the same load, different stress.

Figure 5.10 Shear stress between two load blocks.

Figure 5.3 Column load.
**Bending Stress**

- normal stress caused by bending

\[ f_b = \frac{Mc}{I} = \frac{M}{S} \]

**Torsional Stress**

- shear stress caused by twisting

\[ f_v = \frac{T\rho}{J} \]

**Structures and Shear**

- what structural elements see shear?
  - beams
  - bolts
  - splices
  - slabs
  - footings
  - walls
    - wind
    - seismic loads

**Bolts**

- connected members in tension cause shear stress

- connected members in compression cause bearing stress
**Single Shear**

- seen when 2 members are connected

\[
f_v = \frac{P}{A} = \frac{P}{\pi \frac{d^2}{4}}
\]

**Double Shear**

- seen when 3 members are connected
- two areas

\[
f_v = \frac{P}{2A} = \frac{P}{2} = \frac{P}{\pi \frac{d^2}{4}}
\]

**Bolt Bearing Stress**

- compression & contact
- projected area

\[
f_p = \frac{P}{A_{projected}} = \frac{P}{td}
\]

**Strain**

- materials deform
- axially loaded materials change length
- bending materials deflect

\[
strain = \varepsilon = \frac{\Delta L}{L}
\]
Shearing Strain
- deformations with shear
- parallelogram
- change in angles
- stress: $\tau$
- strain: $\gamma = \frac{\delta_s}{L} = \tan \phi \cong \phi$
  - unitless (radians)

Shearing Strain
- deformations with torsion
- twist
- change in angle of line
- stress: $\tau$
- strain: $\gamma = \frac{\rho \phi}{L}$
  - unitless (radians)

Load and Deformation
- for stress, need $P$ & $A$
- for strain, need $\delta$ & $L$
  - how?
  - TEST with load and measure
  - plot $P/A$ vs. $\varepsilon$

Material Behavior
- every material has its own response
  - 10,000 psi
  - $L = 10$ in
  - Douglas Fir vs. steel?

Figure 5.20 Stress-strain diagram for various materials.
Behavior Types
• ductile - “necking”
• true stress
  \[ f = \frac{P}{A} \]
• engineering stress
  – (simplified)
  \[ f = \frac{P}{A_0} \]

Stress to Strain
• important to us in \( f - \varepsilon \) diagrams:
  – straight section
  – LINEAR-ELASTIC
  – recovers shape (no permanent deformation)

Hooke’s Law
• straight line has constant slope
• Hooke’s Law
  \[ f = E \cdot \varepsilon \]
• \( E \)
  – Modulus of elasticity
  – Young’s modulus
  – units just like stress
Stiffness

- ability to resist strain

- steels
  - same E
  - different yield points
  - different ultimate strength

Isotropy & Anisotropy

- **ISOTROPIC**
  - materials with E same at any direction of loading
  - ex. steel

- **ANISOTROPIC**
  - materials with different E at any direction of loading
  - ex. wood is orthotropic

Elastic, Plastic, Fatigue

- elastic springs back
- plastic has permanent deformation
- fatigue caused by reversed loading cycles

Plastic Behavior

- ductile

Figure 5.20  Stress-strain diagram for various materials.

Figure 5.22  Stress-strain diagram for mild steel (A36) with key points highlighted.
**Lateral Strain**

- or “what happens to the cross section with axial stress”

\[ \varepsilon_x = \frac{f_x}{E} \]

\[ f_y = f_z = 0 \]

- strain in lateral direction
  - negative
  - equal for isometric materials \[ \varepsilon_y = \varepsilon_z \]

**Poisson’s Ratio**

- constant relationship between longitudinal strain and lateral strain

\[ \mu = -\frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x} \]

\[ \varepsilon_y = \varepsilon_z = -\frac{\mu f_x}{E} \]

- sign! \( 0 < \mu < 0.5 \)

**Calculating Strain**

- from Hooke’s law

\[ f = E \cdot \varepsilon \]

- substitute

\[ \frac{P}{A} = E \cdot \frac{\delta}{L} \]

- get \( \Rightarrow \)

\[ \delta = \frac{PL}{AE} \]

**Orthotropic Materials**

- non-isometric
- directional values of \( E \) and \( \mu \)
- ex:
  - plywood
  - laminates
  - polymer composites
Stress Concentrations

- why we use $f_{ave}$
- increase in stress at changes in geometry
  - sharp notches
  - holes
  - corners

Maximum Stresses

- if we need to know where $f_{max}$ and $f_v$ happen:

\[
\theta = 0^\circ \rightarrow \cos \theta = 1 \quad f_{max} = \frac{P}{A_o}
\]
\[
\theta = 45^\circ \rightarrow \cos \theta = \sin \theta = \sqrt{0.5} \quad f_{v\text{-max}} = \frac{P}{2A_o} = \frac{f_{max}}{2}
\]

Deformation Relationships

- physical movement
  - axially (same or zero)
  - rotations from axial changes

\[
\delta = \frac{PL}{AE} \quad \text{relates } \delta \text{ to } P
\]
Deformations from Temperature

- atomic chemistry reacts to changes in energy
- solid materials
  - can contract with decrease in temperature
  - can expand with increase in temperature
- linear change can be measured per degree

Thermal Deformation

- \( \alpha \) - the rate of strain per degree
- UNITS: \( \degree F \), \( \degree C \)
- length change: \( \delta_T = \alpha(\Delta T)L \)
- thermal strain: \( \varepsilon_T = \alpha(\Delta T) \)
  - no stress when movement allowed

Coefficients of Thermal Expansion

<table>
<thead>
<tr>
<th>Material</th>
<th>Coefficients (( \alpha )) [in./in./( \degree )F]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood</td>
<td>3.0 x 10^{-6}</td>
</tr>
<tr>
<td>Glass</td>
<td>4.4 x 10^{-6}</td>
</tr>
<tr>
<td>Concrete</td>
<td>5.5 x 10^{-6}</td>
</tr>
<tr>
<td>Cast Iron</td>
<td>5.9 x 10^{-6}</td>
</tr>
<tr>
<td>Steel</td>
<td>6.5 x 10^{-6}</td>
</tr>
<tr>
<td>Wrought Iron</td>
<td>6.7 x 10^{-6}</td>
</tr>
<tr>
<td>Copper</td>
<td>9.3 x 10^{-6}</td>
</tr>
<tr>
<td>Bronze</td>
<td>10.1 x 10^{-6}</td>
</tr>
<tr>
<td>Brass</td>
<td>10.4 x 10^{-6}</td>
</tr>
<tr>
<td>Aluminum</td>
<td>12.8 x 10^{-6}</td>
</tr>
</tbody>
</table>

Stresses and Thermal Strains

- if thermal movement is restrained stresses are induced
  1. bar pushes on supports
  2. support pushes back
  3. reaction causes internal stress
    \[
    f = \frac{P}{A} = \frac{\delta}{L} E
    \]
**Superposition Method**

- can remove a support to make it look determinant
- replace the support with a reaction
- enforce the geometry constraint

\[
\delta_p = -\frac{PL}{AE} \\
\delta_T = \alpha(\Delta T)L
\]

\[
f = -\frac{P}{A} = -\alpha(\Delta T)E
\]

**Design of Members**

- beyond allowable stress...
- materials aren’t uniform 100% of the time
  - ultimate strength or capacity to failure may be different and some strengths hard to test for

**Factor of Safety**

- accommodate uncertainty with a safety factor:
  \[
  \text{allowable load} = \frac{\text{ultimate load}}{F.S}
  \]

- with linear relation between load and stress:
  \[
  F.S = \frac{\text{ultimate load}}{\text{allowable load}} = \frac{\text{ultimate stress}}{\text{allowable stress}}
  \]

\[
f_u = \frac{P_u}{A}
\]
Load and Resistance Factor Design

- loads on structures are
  - not constant
  - can be more influential on failure
  - happen more or less often
  - UNCERTAINTY

\[
R_u = \gamma_D R_D + \gamma_L R_L \leq \phi R_n
\]

\(\phi\) - resistance factor
\(\gamma\) - load factor for (D)ead & (L)ive load