Center of Gravity

- location of equivalent weight
- determined with calculus

\[
\sum \Delta W = \int dW
\]

Centroid

- “average” x & y of an area
- for a volume of constant thickness
  - \( \Delta W = \gamma \Delta A \) where \( \gamma \) is weight/volume
  - center of gravity = centroid of area

\[
\bar{x} = \frac{\sum (x\Delta A)}{A}
\]
\[
\bar{y} = \frac{\sum (y\Delta A)}{A}
\]
**Centroid**

- for a line, sum up length

\[ \bar{x} = \frac{\sum (x \Delta L)}{L} \]

\[ \bar{y} = \frac{\sum (y \Delta L)}{L} \]

**1st Moment Area**

- math concept
- the moment of an area about an axis

\[ Q_x = \bar{y}A \]

\[ Q_y = \bar{x}A \]

**Symmetric Areas**

- symmetric about an axis

- symmetric about a center point

- mirrored symmetry

**Composite Areas**

- made up of basic shapes
- areas can be negative
- (centroids can be negative for any area)
Basic Procedure
1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table
5. Fill in table
6. Sum necessary columns
7. Calculate $\bar{x}$ and $\bar{y}$

Area Centroids
• Table 7.1 – pg. 242

Moments of Inertia
• 2nd moment area
  – math concept
  – area $x$ (distance)$^2$
• need for behavior of
  – beams
  – columns

Moment of Inertia
• about any reference axis
• can be negative

\[ I_y = \int x^2 \, dA \]
\[ I_x = \int y^2 \, dA \]
• resistance to bending and buckling
**Moment of Inertia**

- same area moved away a distance
  - larger $I$

**Polar Moment of Inertia**

- for roundish shapes
- uses polar coordinates ($r$ and $\theta$)
- resistance to twisting

$$ J_o = \int r^2 dA $$

**Radius of Gyration**

- measure of inertia with respect to area

$$ r_x = \sqrt{\frac{I_x}{A}} $$

**Parallel Axis Theorem**

- can find composite $I$ once composite centroid is known (basic shapes)

$$ I_x = I_{cx} + Ad_y^2 $$
$$ = \bar{I}_x + Ad_y^2 $$

$$ I = \sum \bar{I} + \sum Ad^2 $$

$$ \bar{I} = I - Ad^2 $$
Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table with $A, \bar{x}, \bar{A}, \bar{y}, \bar{y}A, \bar{I}$'s, $d$'s, and $A d^2$'s
5. Fill in table and get $\hat{x}$ and $\hat{y}$ for composite
6. Sum necessary columns
7. Sum $\bar{I}$'s and $A d^2$'s

\[
\begin{align*}
\bar{x} &= \frac{\int x A \, dx}{\int A \\
\bar{y} &= \frac{\int y A \, dx}{\int A}
\end{align*}
\]

Area Moments of Inertia

- Table 7.2 – pg. 252: (bars refer to centroid)
  - $x, y$
  - $x', y'$
  - $C$

\[
\begin{align*}
&\text{Rectangle} \\
&\bar{I}_x = \frac{1}{12}bh^3 \\
&\bar{I}_y = \frac{1}{12}b^3h
\end{align*}
\]

\[
\begin{align*}
&\text{Triangle} \\
&\bar{I}_x = \frac{1}{48}bh^3 \\
&\bar{I}_y = \frac{1}{48}b^3h
\end{align*}
\]

\[
\begin{align*}
&\text{Circle} \\
&\bar{I}_x = \frac{1}{4}\pi r^4 \\
&\bar{I}_y = \frac{1}{4}\pi r^4
\end{align*}
\]