beams: bending and shear stress
Beam Bending

• Galileo
  – relationship between stress and depth$^2$

• can see
  – top squishing
  – bottom stretching

• what are the stress across the section?
Pure Bending

- bending only
- no shear
- axial normal stresses from bending can be found in
  - homogeneous materials
  - plane of symmetry
  - follow Hooke’s law
Bending Moments

• **sign convention:**

  
  ![Diagram showing positive and negative bending moments](image)

  + ![Positive moment](image)

  - ![Negative moment](image)

• **size of maximum internal moment will govern our design of the section**
Normal Stresses

• geometric fit
  – plane sections remain plane
  – stress varies linearly
Neutral Axis

• stresses vary linearly

• zero stress occurs at the centroid

• neutral axis is line of centroids (n.a.)
Derivation of Stress from Strain

- pure bending = arc shape

\[ L = R \theta \]

\[ L_{outside} = (R + y)\theta \]

\[ \varepsilon = \frac{\delta}{L} = \frac{L_{outside} - L}{L} = \frac{(R + y)\theta - R\theta}{R\theta} = \frac{y}{R} \]
Derivation of Stress

- zero stress at n.a.

\[ f = E \varepsilon = \frac{E_y}{R} \]

\[ f_{\text{max}} = \frac{E_c}{R} \]

\[ f = \frac{y}{c} f_{\text{max}} \]
Bending Moment

- resultant moment from stresses = bending moment!

\[ M = \Sigma f y \Delta A \]

\[ = \Sigma \frac{y f_{\text{max}}}{c} y \Delta A = \frac{f_{\text{max}}}{c} \Sigma y^2 \Delta A = \frac{f_{\text{max}}}{c} I = f_{\text{max}} S \]
Bending Stress Relations

\[ \frac{1}{R} = \frac{M}{EI} \]
\[ f_b = \frac{My}{I} \]
\[ S = \frac{I}{c} \]

curvature  
general bending stress  
section modulus

\[ f_b = \frac{M}{S} \]
\[ S_{\text{required}} \geq \frac{M}{F_b} \]

maximum bending stress  
required section modulus for design
Transverse Loading and Shear

- perpendicular loading
- internal shear
- along with bending moment
Bending vs. Shear in Design

- bending stresses dominate
- shear stresses exist horizontally with shear
- no shear stresses with pure bending
Shear Stresses

- horizontal & vertical
Shear Stresses

• horizontal & vertical
Beam Stresses

- horizontal with bending
Equilibrium

• horizontal force \( V \) needed

\[
V_{\text{longitudinal}} = \frac{V_T Q}{I} \Delta x
\]

• \( Q \) is a moment area
Moment of Area

- Q is a moment area with respect to the n.a. of area above or below the horizontal

- $Q_{\text{max}}$ at $y=0$ (neutral axis)

- q is shear flow:
  
  $$ q = \frac{V_{\text{longitudinal}}}{\Delta x} = \frac{V_T Q}{I} $$
Shearing Stresses

\[ f_v = \frac{V}{\Delta A} = \frac{V}{b \cdot \Delta x} \]

\[ f_{v-\text{ave}} = \frac{VQ}{Ib} \]

- \( f_v = 0 \) on the top/bottom
- \( b \) min may not be with \( Q \) max
- with \( h/4 \geq b \), \( f_{v-\text{max}} \leq 1.008 f_{v-\text{ave}} \)
Rectangular Sections

\[ I = \frac{bh^3}{12} \quad Q = A\overline{y} = \frac{bh^2}{8} \]

\[ f_v = \frac{VQ}{Ib} = \frac{3V}{2A} \]

- \( f_{v-max} \) occurs at n.a.
Steel Beam Webs

- **W and S sections**
  - $b$ varies
  - stress in flange negligible
  - presume constant stress in web

\[
\sigma_{v,\text{max}} = \frac{3V}{2A} \approx \frac{V}{A_{\text{web}}}
\]
Shear Flow

- loads applied in plane of symmetry
- cut made perpendicular

\[ q = \frac{VQ}{I} \]
Shear Flow Quantity

• sketch from $Q$

$$q = \frac{VQ}{I}$$
Connectors Resisting Shear

- plates with
  - nails
  - rivets
  - bolts
- splices

\[
\frac{V_{\text{longitudinal}}}{p} = \frac{VQ}{I} \geq \frac{VQ_{\text{connected area}}}{I} \cdot p
\]
Vertical Connectors

- *isolate an area with vertical interfaces*

\[ nF_{\text{connector}} \geq \frac{VQ_{\text{connected area}}}{I} \cdot p \]
Unsymmetrical Shear or Section

- **member can bend and twist**
  - not symmetric
  - shear not in that plane
- **shear center**
  - moments balance