Rigid frames: compression & buckling
Rigid Frames

- rigid frames have no pins
- frame is all one body
- joints transfer moments and shear
- typically statically indeterminate
- types
  - portal
  - gable
Rigid Frames

• behavior
Rigid Frames

- moments get redistributed
- deflections are smaller
- effective column lengths are shorter
- very sensitive to settling
Moment Redistribution

- continuous slabs & beams with uniform loading
  - joints similar to fixed ends, but can rotate
- change in moment to center $= \frac{wL^2}{8}$
  - $M_{\text{max}}$ for simply supported beam
Rigid Frames

- resists lateral loadings
- shape depends on stiffness of beams and columns
- $90^\circ$ maintained
Rigid Frames

- staggered truss
  - rigidity
  - clear stories
Rigid Frames

- connections
  - steel
  - concrete

Picture of a rigid frame structure with labels "2M4" and "Fixed".
Braced Frames

- pin connections
- bracing to prevent lateral movements

http://nisee.berkeley.edu/godden
Braced Frames

- types of bracing
  - knee-bracing
  - diagonal
  - X
  - K or chevron
  - shear walls
Shear Walls

- resist lateral load in plane with wall
Compression Members

- designed for strength & stresses
- designed for serviceability & deflection
- need to design for stability
  - ability to support a specified load without sudden or unacceptable deformations
Column Buckling

- axially loaded columns
- long & slender
  - unstable equilibrium = buckling
  - sudden and not good
Modeling

• can be modeled with a spring at mid-height
• when moment from deflection exceeds the spring capacity ... “boing”
• critical load $P$
Effect of Length

- long & slender
- short & stubby
Buckling Load

- related to deflected shape \( (P\Delta) \)
- shape of sine wave
- Euler’s Formula
- smallest \( I \) governs

\[
P_{\text{critical}} = \frac{\pi^2 EI}{(L)^2}
\]

Figure 9.3  Leonhard Euler (1707–1783).
Critical Stress

- short columns

\[ f_{\text{critical}} = \frac{P_{\text{actual}}}{A} < F_a \]

- slenderness ratio = \( L_e/r \) (L/d)

- radius of gyration = \( r = \sqrt{\frac{I}{A}} \)

\[ f_{\text{critical}} = \frac{P_{\text{critical}}}{A} = \frac{\pi^2 E A r^2}{A (L_e)^2} = \frac{\pi^2 E}{(L_e/r)^2} \]

\[ P_{\text{critical}} = \frac{\pi^2 E A}{(L_e/r)^2} \]
Critical Stresses

- when a column gets stubby, $F_y$ will limit the load
- real world has loads with eccentricity
- $C_c$ for steel and allowable stress

$$\frac{L_e}{r} > C_c = \sqrt{\frac{2\pi^2 E}{F_y}}$$

Fig. 10.9
Effective Length

- end conditions affect shape
- effective length factor, \( K \), \( L_e = K \cdot L \)

<table>
<thead>
<tr>
<th>Buckled shape of column shown by dashed line</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theoretical ( K ) value</td>
<td>0.5</td>
<td>0.7</td>
<td>1.0</td>
<td>1.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Recommended design values when ideal conditions are approximated</td>
<td>0.65</td>
<td>0.80</td>
<td>1.0</td>
<td>1.2</td>
<td>2.10</td>
<td>2.0</td>
</tr>
</tbody>
</table>
Bracing

- bracing affects shape of buckle in one direction
- both should be checked!
Centric & Eccentric Loading

- **centric**
  - allowable stress from strength or buckling
- **eccentric**
  - combined stresses
Combined Stresses

- axial + bending

\[ f_{\text{max}} = \frac{P}{A} + \frac{Mc}{I} \]

\[ M = P \cdot e \]

- design

\[ f_{\text{max}} \leq F_{cr} = \frac{f_{cr}}{F.S.} \]
Stress Limit Conditions

– ASD interaction formula

\[
\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0
\]

– with biaxial bending

\[
\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0
\]

interaction diagram
Stress Limit Conditions

- in reality, as the column flexes, the moment increases

- $P-\Delta$ effect

$$\frac{f_a}{F_a} + \frac{f_b \times (\text{Magnification factor})}{F_{bx}} \leq 1.0$$
Rigid Frame Analysis

- members see
  - shear
  - axial force
  - bending

- V & M diagrams
  - plot on “outside”
Rigid Frame Analysis

- need support reactions
- free body diagram each member
- end reactions are equal and opposite on next member
- “turn” member like beam
- draw V & M
Rigid Frame Analysis

– FBD & M

• opposite end reactions at joints
Rigid Frame Design

- **loads and combinations**
  - usually uniformly distributed gravity loads
  - worst case for largest moments...
  - wind direction can increase moments
Rigid Frame Design

- frames & floors
  - rigid frame can have slab floors or slab with connecting beams

- other
  - slabs or plates on columns
Rigid Frame Design

- **floors – plates & slabs**
  - one-way behavior
    - side ratio > 1.5
    - “strip” beam
  - two-way behavior
    - more complex
Rigid Frame Design

- **columns in frames**
  - ends can be “flexible”
  - stiffness affected by beams and column = $EI/L$

$$G = \Psi = \frac{\sum EI}{\sum EI} \frac{l_c}{l_b}$$

- for the joint
  - $l_c$ is the column length of each column
  - $l_b$ is the beam length of each beam
  - measured center to center
Rigid Frame Design

- column effective length, $k$
Tools – Multiframe

• in computer lab
Tools – Multiframe

- **frame window**
  - define frame members
    - or pre-defined frame
  - select points, assign supports
  - select members, assign section
  - load window
  - select point or member, add point or distributed loads
Tools – Multiframe

• to run analysis choose
  – Analyze menu
    • Linear

• plot
  – choose options

• results
  – choose options