Concrete Beam Design

- composite of concrete and steel
- American Concrete Institute (ACI)
  - design for maximum stresses
  - limit state design
    - service loads x load factors
    - concrete holds no tension
    - failure criteria is yield of reinforcement
    - failure capacity x reduction factor
    - factored loads < reduced capacity
  - concrete strength = $f'_c$

Concrete Construction

- cast-in-place
- tilt-up
- prestressing
- post-tensioning

Concrete Beams

- types
  - reinforced
  - precast
  - prestressed
- shapes
  - rectangular, I
  - T, double T’s, bulb T’s
  - box
  - spandrel
Concrete Beams

- shear
  - vertical
  - horizontal
  - combination:
    - tensile stresses at 45°

- bearing
  - crushing

Concrete

- low strength to weight ratio
- relatively inexpensive
  - Portland cement
    - types I - V
  - aggregate
    - course & fine
  - water
  - admixtures
    - air entraining
    - superplasticizers

Concrete

- hydration
  - chemical reaction
  - workability
  - water to cement ratio
  - mix design
- fire resistant
- cover for steel
- creep & shrinkage

Concrete

- placement (not pouring!)
- vibrating
- screeding
- floating
- troweling
- curing
- finishing
Reinforcement

• deformed steel bars (rebar)
  – Grade 40, \( F_y = 40 \text{ ksi} \)
  – Grade 60, \( F_y = 60 \text{ ksi} \) - most common
  – Grade 75, \( F_y = 75 \text{ ksi} \)
  – US customary in # of 1/8” \( \phi \)
• longitudinally placed
  – bottom
  – top for compression reinforcement

Composite Beams

• concrete
  – in compression
• steel
  – in tension
• shear studs

Reinforcement

• prestressing strand
• post-tensioning
• stirrups
• detailing
  – development length
  – anchorage
  – splices

Behavior of Composite Members

• plane sections remain plane
• stress distribution changes

\[ f_1 = E_1 \epsilon = -\frac{E_1 y}{R} \]
\[ f_2 = E_2 \epsilon = -\frac{E_2 y}{R} \]
Transformation of Material

- $n$ is the ratio of $E$'s
  \[ n = \frac{E_2}{E_1} \]

- effectively widens a material to get same stress distribution

Stresses in Composite Section

- with a section transformed to one material, new $I$

  - stresses in that material are determined as usual
  - stresses in the other material need to be adjusted by $n$

\[
\frac{E_2}{E_1} = \frac{E_{\text{steel}}}{E_{\text{concrete}}}
\]

\[
f_c = -\frac{My}{I_{\text{transformed}}}
\]

\[
f_s = -\frac{Myn}{I_{\text{transformed}}}
\]

Reinforced Concrete - stress/strain

- for stress calculations
  - steel is transformed to concrete
  - concrete is in compression above n.a. and represented by an equivalent stress block
  - concrete takes no tension
  - steel takes tension
  - force ductile failure
Location of n.a.

- Ignore concrete below n.a.
- Transform steel
- Same area moments, solve for \( x \)

\[
 bx \cdot \frac{x}{2} - nA_s (d - x) = 0
\]

T sections

- N.a. equation is different if n.a. below flange

\[
 b_r h_f \left( x - \frac{h_f}{2} \right) + (x - h_f) b_w \left( x - h_f \right) \frac{1}{2} - nA_s (d - x) = 0
\]

ACI Load Combinations*

- \( 1.4D \)
- \( 1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R) \)
- \( 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W) \)
- \( 1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R) \)
- \( 1.2D + 1.0E + 1.0L + 0.2S \)
- \( 0.9D + 1.0W \)
- \( 0.9D + 1.0E \)

*can also use old ACI factors

Reinforced Concrete Design

- Stress distribution in bending

\[
\beta_c = \frac{0.85f'_c}{a/2}
\]

actual stress

Whitney stress block

Wang & Salmon, Chapter 3
Force Equations

- \( C = 0.85 f'_c ba \)
- \( T = A_s f_y \)
- where
  - \( f'_c \) = concrete compressive strength
  - \( a \) = height of stress block
  - \( \beta_1 \) = factor based on \( f'_c \)
  - \( c \) = location to the n.a.
  - \( b \) = width of stress block
  - \( f_y \) = steel yield strength
  - \( A_s \) = area of steel reinforcement

Equilibrium

- \( T = C \)
- \( M_n = T(d-a/2) \)
  - \( d \) = depth to the steel n.a.
- with \( A_s \)
  - \( a = \frac{A_s f_y}{0.85 f'_c b} \)
  - \( \phi = 0.65 + (\varepsilon_{s} - \varepsilon_{c}) \frac{0.25}{(0.005 - \varepsilon_{c})} \geq 0.65 \)
  - \( M_u \leq \phi M_n \)
  - \( \phi M_n = \phi T(d-a/2) = A_s f_y (d-a/2) \)

Over and Under-reinforcement

- over-reinforced
  - steel won't yield
- under-reinforced
  - steel will yield
- reinforcement ratio
  - \( \rho = \frac{A_s}{bd} \)
  - use as a design estimate to find \( A_s,b,d \)
  - max \( \rho \) is found with \( \varepsilon_{\text{steel}} \geq 0.004 \) (not \( \rho_{\text{bal}} \))
  - *with \( \varepsilon_{\text{steel}} \geq 0.005, \phi = 0.9 \)

\[ \text{http://people.bath.ac.uk/abstji/concrete_video/virtual_lab.htm} \]

\( A_s \) for a Given Section

- several methods
  - guess \( a \) and iterate
    1. guess \( a \) (less than n.a.)
    2. \( A_s = \frac{0.85 f'_c ba}{f_y} \)
    3. solve for \( a \) from \( M_u = \phi A_s f_y (d-a/2) \)
    \[ a = 2 \left( d - \frac{M_u}{\phi A_s f_y} \right) \]
    4. repeat from 2. until \( a \) from 3. matches \( a \) in 2.
**A_s for a Given Section (cont)**

- **chart method**
  - Wang & Salmon Fig. 3.8.1 \( R_n \) vs. \( \rho \)
    1. calculate \( R_n = \frac{M_n}{bd^2} \)
    2. find curve for \( f_c' \) and \( f_y \) to get \( \rho \)
    3. calculate \( A_s \) and \( a \)
- **simplify by setting** \( h = 1.1d \)

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**Reinforcement**

- **min for crack control**
- **required**
  \[ A_s = \frac{3 \sqrt{f_c'}}{f_y} (bd) \]
- **not less than**
  \[ A_s = \frac{200}{f_y} (bd) \]
- **A_{s-max}**
  \[ a = \beta_1 (0.375d) \]
- **typical cover**
  - 1.5 in, 3 in with soil
- **bar spacing**

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**Shells**

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**Annunciation Greek Orthodox Church**

- **Wright, 1956**
Annunciation Greek Orthodox Church
• Wright, 1956

Cylindrical Shells
• can resist tension
• shape adds “depth”
• not vaults
• barrel shells

Kimball Museum, Kahn 1972
• outer shell edges
Kimball Museum, Kahn 1972

- skylights at peak

Approximate Depths