Concrete Beam Design

- composite of concrete and steel
- American Concrete Institute (ACI)
  - design for maximum stresses
  - limit state design
    - service loads x load factors
    - concrete holds no tension
    - failure criteria is yield of reinforcement
    - failure capacity x reduction factor
    - factored loads < reduced capacity
  - concrete strength = $f'_c$

Concrete Construction

- cast-in-place
- tilt-up
- prestressing
- post-tensioning

Concrete Beams

- types
  - reinforced
  - precast
  - prestressed
- shapes
  - rectangular, I
  - T, double T's, bulb T's
  - box
  - spandrel
Concrete Beams

• shear
  – vertical
  – horizontal
  – combination:
    • tensile stresses at 45°

• bearing
  – crushing

Concrete

• hydration
  – chemical reaction
  – workability
  – water to cement ratio
  – mix design

• fire resistant
• cover for steel
• creep & shrinkage

Concrete

• low strength to weight ratio
• relatively inexpensive
  – Portland cement
    • types I - V
  – aggregate
    • course & fine
  – water
  – admixtures
    • air entraining
    • superplasticizers

Concrete

• placement (not pouring!)
• vibrating
• screeding
• floating
• troweling
• curing
• finishing
Reinforcement

- deformed steel bars (rebar)
  - Grade 40, $F_y = 40$ ksi
  - Grade 60, $F_y = 60$ ksi - most common
  - Grade 75, $F_y = 75$ ksi
  - US customary in # of 1/8” $\phi$

- longitudinally placed
  - bottom
  - top for compression reinforcement

Composite Beams

- concrete
  - in compression
- steel
  - in tension
- shear studs

Reinforcement

- prestressing strand
- post-tensioning
- stirrups
- detailing
  - development length
  - anchorage
  - splices

Behavior of Composite Members

- plane sections remain plane
- stress distribution changes

$$f_1 = E_1 \varepsilon = \frac{- E_1 y}{R}$$
$$f_2 = E_2 \varepsilon = \frac{- E_2 y}{R}$$

http://nisee.berkeley.edu/godden
Transformation of Material

- \( n = \frac{E_2}{E_1} \) is the ratio of E's
- effectively widens a material to get same stress distribution

Stresses in Composite Section

- with a section transformed to one material, new I
  - stresses in that material are determined as usual
  - stresses in the other material need to be adjusted by \( n \)

\[
E_n = \frac{E_{\text{steel}}}{E_{\text{concrete}}}
\]

Reinforced Concrete - stress/strain

- for stress calculations
  - steel is transformed to concrete
  - concrete is in compression above n.a. and represented by an equivalent stress block
  - concrete takes no tension
  - steel takes tension
  - force ductile failure
Location of n.a.
• ignore concrete below n.a.
• transform steel
• same area moments, solve for $x$

$$b x \cdot \frac{x}{2} - nA_s (d - x) = 0$$

T sections
• n.a. equation is different if n.a. below flange

$$b_r h_r \left( x - \frac{h_r}{2} \right) + (x - h_r) b_w \left( \frac{x - h_r}{2} \right) - nA_s (d - x) = 0$$

ACI Load Combinations*
• $1.4D$
• $1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$
• $1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W)$
• $1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$
• $1.2D + 1.0E + 1.0L + 0.2S$
• $0.9D + 1.0W$
• $0.9D + 1.0E$

*can also use old ACI factors

Reinforced Concrete Design
• stress distribution in bending

Wang & Salmon, Chapter 3
Force Equations

- \( C = 0.85 f'_c ba \)
- \( T = A_s f_y \)
- where
  - \( f'_c \) = concrete compressive strength
  - \( a \) = height of stress block
  - \( \beta_i \) = factor based on \( f'_c \)
  - \( c \) = location to the n.a.
  - \( b \) = width of stress block
  - \( f_y \) = steel yield strength
  - \( A_s \) = area of steel reinforcement

Equilibrium

- \( T = C \)
- \( M_n = T(d-a/2) \)
  - \( d \) = depth to the steel n.a.
- with \( A_s \)
  - \( a = \frac{A_s f_y}{0.85 f'_c b} \)
  - \( \phi = 0.65 + (\varepsilon_u - \varepsilon_y) \frac{0.25}{(0.005 - \varepsilon_y)} \geq 0.65 \)
  - \( M_u \leq \phi M_n \)
  - \( \phi M_n = \phi T(d-a/2) = A_s f_y (d-a/2) \)

Over and Under-reinforcement

- over-reinforced
  - steel won’t yield
- under-reinforced
  - steel will yield
- reinforcement ratio
  - \( \rho = \frac{A_s}{bd} \)
  - use as a design estimate to find \( A_s, b, d \)
  - max \( \rho \) is found with \( \varepsilon_{steel} \geq 0.004 \) (not \( \rho_{bal} \))
  - *with \( \varepsilon_{steel} \geq 0.005, \phi = 0.9 \)

\[ A_s \] for a Given Section

- several methods
  - guess \( a \) and iterate
    1. guess \( a \) (less than n.a.)
    2. \( a = \frac{A_s f_y}{0.85 f'_c b} \)
    3. solve for \( a \) from \( M_u = A_s f_y (d-a/2) \)
    \[ a = 2 \left( d - \frac{M_u}{\phi A_s f_y} \right) \]
  4. repeat from 2. until \( a \) from 3. matches \( a \) in 2.
**A_s for a Given Section (cont)**

- **chart method**
  - Wang & Salmon Fig. 3.8.1  \( R_n \) vs. \( \rho \)
  1. calculate \( R_n = \frac{M_n}{bd^2} \)
  2. find curve for \( f'c \) and \( f_y \) to get \( \rho \)
  3. calculate \( A_s \) and \( a \)
- **simplify by setting** \( h = 1.1d \)

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**Reinforcement**

- **min for crack control**
- **required**
  \[
  A_s = \frac{3\sqrt{f'c}}{f_y} (bd)
  \]
- **not less than**
  \[
  A_s = \frac{200}{f_y} (bd)
  \]
- **A_s-max** : \( a = \beta_i (0.375d) \)
- **typical cover**
  - 1.5 in, 3 in with soil
- **bar spacing**

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**Shells**

**Annunciation Greek Orthodox Church**

- **Wright, 1956**
Annunciation Greek Orthodox Church

- Wright, 1956

Cylindrical Shells

- can resist tension
- shape adds “depth”

Kimball Museum, Kahn 1972

- outer shell edges
Kimball Museum, Kahn 1972

- skylights at peak

Approximate Depths

<table>
<thead>
<tr>
<th>Beams (placed in plant)</th>
<th>Typical span length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete Beam</td>
<td>L30-L40</td>
</tr>
<tr>
<td>Steel Beam</td>
<td>L30-L40</td>
</tr>
<tr>
<td>Timber Beam</td>
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</tr>
<tr>
<td>Two-way beam</td>
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<tr>
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<tr>
<td>Flat plate</td>
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<tr>
<td>Minimum span</td>
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<td>Possible span range</td>
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- Multiplane
- Flat plate
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