Shear in Concrete Beams

- flexure combines with shear to form diagonal cracks
- horizontal reinforcement doesn’t help
- stirrups = vertical reinforcement

ACI Shear Values

- shear stress (beams)
  - \( \sigma_v = \frac{2}{\phi} \sqrt{f'_c} \quad f'_c \text{ is in psi} \)
  - \( \lambda \) for lightweight mat’ls
- shear strength:
  - \( V_u \leq \phi V_c + \phi V_s \)
  - \( V_s \) is strength from stirrup reinforcement
Concrete Shear  5
Lecture 24
Foundations Structures
ARCH 331
F2008abn
Stirrup Reinforcement
• shear capacity:
\[ V_s = \frac{A_v f_y d}{s} \]
– \( A_v \) = area in all legs of stirrups
– \( s \) = spacing of stirrups
• may need stirrups when concrete has enough strength!

Required Stirrup Reinforcement
• spacing limits

Torsional Stress & Strain
• can see torsional stresses & twisting of axi-symmetrical cross sections
– torque
– remain plane
– undistorted
– rotates
• not true for square sections....

Shear Stress Distribution
• depend on the deformation
• \( \phi \) = angle of twist
– measure
• can prove planar section doesn’t distort
Shearing Strain

• related to $\phi$ 
  \[ \gamma = \frac{\rho \phi}{L} \]

• $\rho$ is the radial distance from the centroid to the point under strain

• shear strain varies linearly along the radius: $\gamma_{\text{max}}$ is at outer diameter

Torsional Stress - Strain

• know $f_v = \tau = G \cdot \gamma$ and $\gamma = \frac{\rho \phi}{L}$

• so 
  \[ \tau = G \cdot \frac{\rho \phi}{L} \]

• where $G$ is the Shear Modulus

Shear Stress

• $\tau_{\text{max}}$ happens at outer diameter

• combined shear and axial stresses
  – maximum shear stress at 45° “twisted” plane
Shear Strain

• knowing $\tau = G \cdot \frac{\rho \phi}{L}$ and $\tau = \frac{T \rho}{J}$

• solve: $\phi = \frac{TL}{JG}$

• composite shafts: $\phi = \sum_i \frac{T_i L_i}{J_i G_i}$

Noncircular Shapes

• torsion depends on $J$

• plane sections don’t remain plane

• $\tau_{\text{max}}$ is still at outer diameter

• where $a$ is longer side ($> b$)

Open Thin-Walled Sections

• with very large $a/b$ ratios:

\[
\tau_{\text{max}} = \frac{T}{\frac{1}{3} ab^2} \quad \phi = \frac{TL}{\frac{1}{3} ab^3 G}
\]

Shear Flow in Closed Sections

• $q$ is the internal shear force/unit length

\[
\tau = \frac{T}{2tA} \quad \phi = \frac{TL}{\frac{4}{3} tA^2} \sum_i \frac{s_i}{t_i}
\]

• $A$ is the area bounded by the centerline

• $s_i$ is the length segment, $t_i$ is the thickness

TABLE 3.1. Coefficients for Rectangular Bars in Torsion

<table>
<thead>
<tr>
<th>a/b</th>
<th>$c_1$</th>
<th>$c_2$</th>
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<tbody>
<tr>
<td>1.0</td>
<td>0.206</td>
<td>0.1406</td>
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<tr>
<td>1.2</td>
<td>0.219</td>
<td>0.1661</td>
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<td>1.5</td>
<td>0.231</td>
<td>0.1958</td>
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<td>2.0</td>
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<tr>
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<tr>
<td>4.0</td>
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<td>0.2810</td>
</tr>
<tr>
<td>5.0</td>
<td>0.291</td>
<td>0.2910</td>
</tr>
<tr>
<td>10.0</td>
<td>0.312</td>
<td>0.3120</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.333</td>
<td>0.3330</td>
</tr>
</tbody>
</table>
Shear Flow in Open Sections

- each segment has proportion of $T$ with respect to torsional rigidity,

$$\tau_{\text{max}} = \frac{T t_{\text{max}}}{\frac{1}{3} \sum b_i t_i^3}$$

- total angle of twist:

$$\phi = \frac{TL}{\frac{1}{3} G \sum b_i t_i^3}$$

- I beams - web is thicker, so $\tau_{\text{max}}$ is in web

Torsional Shear Stress

- twisting moment
- and beam shear

Torsional Shear Reinforcement

- closed stirrups
- more longitudinal reinforcement
- area enclosed by shear flow

Development Lengths

- required to allow steel to yield ($f_y$)
- standard hooks
  - moment at beam end
- splices
  - lapped
  - mechanical connectors
Development Lengths

- \( l_d \), embedment required both sides
- proper cover, spacing:
  - No. 6 or smaller
    \[ l_d = \frac{d_b f_y}{25 \lambda \sqrt{f_c'}} \] or 12 in. minimum
  - No. 7 or larger
    \[ l_d = \frac{d_b f_y}{20 \lambda \sqrt{f_c'}} \] or 12 in. minimum

Development Lengths

- hooks
  - bend and extension
  - \( l_{dh} = \frac{d_b f_y}{50 \lambda \sqrt{f_c'}} \)

Concrete Deflections

- bars in compression
  \[ l_d = \frac{d_b f_y}{50 \lambda \sqrt{f_c'}} \leq 0.0003 f_y d_b \]
- splices
  - tension minimum is function of \( l_d \) and splice classification
  - compression minimum
  - is function of \( d_b \) and \( F_y \)
- elastic range
  - \( E_c \) (with \( f_c' \) in psi)
    - normal weight concrete (~ 145 lb/ft\(^3\))
      \[ E_c = 57,000 \sqrt{f_c'} \]
    - concrete between 90 and 160 lb/ft\(^3\)
      \[ E_c = W_c^{1.5} 33 \sqrt{f_c'} \]
    - cracked
      - I cracked
      - \( E \) adjusted
Deflection Limits

- relate to whether or not beam supports or is attached to a damageable non-structural element
- need to check service live load and long term deflection against these

<table>
<thead>
<tr>
<th>Limit</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>L/180</td>
<td>roof systems (typical) – live</td>
</tr>
<tr>
<td>L/240</td>
<td>floor systems (typical) – live + long term</td>
</tr>
<tr>
<td>L/360</td>
<td>supporting plaster – live</td>
</tr>
<tr>
<td>L/480</td>
<td>supporting masonry – live + long term</td>
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