forces and moments

Structural Math

• **physics takes observable phenomena and relates the measurement with rules:** mathematical relationships

• **need**
  – reference frame
  – measure of length, mass, time, direction, velocity, acceleration, work, heat, electricity, light
  – calculations & geometry

• quantify environmental loads
  – how big is it?

• evaluate geometry and angles
  – where is it?
  – what is the scale?
  – what is the size in a particular direction?

• quantify what happens in the structure
  – how big are the internal forces?
  – how big should the beam be?

Physics for Structures

• measures
  – US customary & SI

<table>
<thead>
<tr>
<th>Units</th>
<th>US</th>
<th>SI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>in, ft, mi</td>
<td>mm, cm, m</td>
</tr>
<tr>
<td>Volume</td>
<td>gallon</td>
<td>liter</td>
</tr>
<tr>
<td>Mass</td>
<td>lb mass</td>
<td>g, kg</td>
</tr>
<tr>
<td>Force</td>
<td>lb force</td>
<td>N, kN</td>
</tr>
<tr>
<td>Temperature</td>
<td>F</td>
<td>C</td>
</tr>
</tbody>
</table>
Physics for Structures

- **scalars** – any quantity
- **vectors** - quantities with direction
  - like displacements
  - summation results in the “straight line path” from start to end
  - **normal vector** is perpendicular to something

![Diagram of vectors and Cartesian coordinates]

Language

- **symbols for operations**: +, -, /, x
- **symbols for relationships**: ( ), =, <, >
- **algorithms**
  - cancellation
  - factors
  - signs
  - ratios and proportions
  - power of a number
  - conversions, ex. \( 1X = 10 \ Y \)
  - operations on both sides of equality

\[
\frac{10Y}{1X} \quad \text{or} \quad \frac{1X}{10Y} = 1
\]

On-line Practice

- eCampus / Study Aids

Geometry

- **angles**
  - right \( = 90^\circ \)
  - acute \( < 90^\circ \)
  - obtuse \( > 90^\circ \)
  - \( \pi = 180^\circ \)
- **triangles**
  - area \( = \frac{b \times h}{2} \)
  - hypotenuse
  - total of angles = \( 180^\circ \)

\[
AB^2 + AC^2 = BC^2
\]
Geometry

- lines and relation to angles
  - parallel lines can’t intersect
  - perpendicular lines cross at 90°
  - intersection of two lines is a point
  - opposite angles are equal when two lines cross

---

Geometry

- intersection of a line with parallel lines results in identical angles
- two lines intersect in the same way, the angles are identical

---

Geometry

- sides of two angles are parallel and intersect opposite way, the angles are supplementary - the sum is 180°
- two angles that sum to 90° are said to be complimentary

---

Geometry

- sides of two angles bisect a right angle (90°), the angles are complimentary
- right angle bisects a straight line, remaining angles are complimentary
### Geometry

- **similar triangles have proportional sides**

\[
\frac{AB}{AD} = \frac{AC}{AE} = \frac{BC}{DE}
\]

![Similar Triangles Diagram](image)

### Trigonometry

- **for right triangles**

\[
\sin \alpha = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{AB}{CB}
\]

\[
\cos \alpha = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{AC}{CB}
\]

\[
\tan \alpha = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{AB}{AC}
\]

**SOHCAHTOA**

### Trigonometry

- **cartesian coordinate system**
  - origin at 0,0
  - coordinates in (x,y) pairs
  - x & y have signs

- **for angles starting at positive x**
  - sin is y side
  - cos is x side

\[
\sin \text{ is y side for 180-360°}
\]

\[
\cos \text{ is x side for 90-270°}
\]

\[
\tan \text{ is y/x for 0-180°}
\]

\[
\tan \text{ is y/x for 270-360°}
\]
Trigonometry

- for all triangles
  - sides A, B & C are opposite angles $\alpha$, $\beta$ & $\gamma$

- LAW of SINES
  \[
  \frac{\sin \alpha}{A} = \frac{\sin \beta}{B} = \frac{\sin \gamma}{C}
  \]

- LAW of COSINES
  \[
  A^2 = B^2 + C^2 - 2BC \cos \alpha
  \]

Algebra

- equations (something = something)
- constants
  - real numbers or shown with a, b, c...
- unknown terms, variables
  - names like R, F, x, y
- linear equations
  - unknown terms have no exponents
- simultaneous equations
  - variable set satisfies all equations

Algebra

- solving one equation
  - only works with one variable
  - ex:
    \[
    2x - 1 = 0
    \]
    - add to both sides
      \[
      2x - 1 + 1 = 0 + 1
      \]
      \[
      2x = 1
      \]
    - divide both sides
      \[
      \frac{2x}{2} = \frac{1}{2}
      \]
      \[
      x = \frac{1}{2}
      \]
Algebra

- solving two equation
  - only works with two variables
  - ex: \(2x + 3y = 8\)
    - look for term similarity \(12x - 3y = 6\)
    - can we add or subtract to eliminate one term?
      - add \(2x + 3y + 12x - 3y = 8 + 6\)
      - get \(x\) by itself on a side \(14x = 14\) \(\frac{14x}{14} = \frac{14}{14} = x = 1\)

Forces

- statics
  - physics of forces and reactions on bodies and systems
  - equilibrium (bodies at rest)

- forces
  - something that exerts on an object:
    - motion
    - tension
    - compression

Forces Characteristics

- applied at a point
- magnitude
  - Imperial units: lb, k (kips)
  - SI units: N (newtons), kN
- direction

- “action of one body on another that affects the state of motion or rest of the body”
- Newton’s 3rd law:
  - for every force of action there is an equal and opposite reaction along the same line

http://www.physics.umd.edu
Forces on Rigid Bodies

- for statics, the bodies are ideally rigid
- can translate and rotate
- internal forces are
  - in bodies
  - between bodies (connections)
- external forces act on bodies

Transmissibility

- the force stays on the same line of action
- truck can’t tell the difference

Forces on Rigid Bodies

- only valid for EXTERNAL forces

Force System Types

- collinear

Force System Types

- coplanar
Force System Types
• space

Adding Vectors
• graphically
  – parallelogram law
    • diagonal
    • long for 3 or more vectors
  – tip-to-tail
    • more convenient with lots of vectors

Force Components
• convenient to resolve into 2 vectors
• at right angles
• in a “nice” coordinate system
• $\theta$ is between $F_x$ and $F$ from $F_x$
  $F_x = F \cos \theta$
  $F_y = F \sin \theta$
  $F = \sqrt{F_x^2 + F_y^2}$
  $\tan \theta = \frac{F_y}{F_x}$

Trigonometry
• $F_x$ is negative
  – 90° to 270°
• $F_y$ is negative
  – 180° to 360°
• $\tan$ is positive
  – quads I & III
• $\tan$ is negative
  – quads II & IV
Component Addition
- find all x components
- find all y components
- find sum of x components, $R_x$ (resultant)
- find sum of y components, $R_y$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$\tan \theta = \frac{R_y}{R_x}$$

Alternative Trig for Components
- doesn’t relate angle to axis direction
- $\phi$ is “small” angle between $F$ and $EITHER F_x$ or $F_y$
- no sign out of calculator!
- have to choose RIGHT trig function, resulting direction (sign) and component axis

Friction
- resistance to movement
- contact surfaces determine $\mu$
- proportion of normal force ($\perp$)
  - opposite to slide direction
  - static > kinetic

$$F = \mu N$$

Cables
- simple
- uses
  - suspension bridges
  - roof structures
  - transmission lines
  - guy wires, etc.
- have same tension all along
- can’t stand compression
Cables Structures
- use high-strength steel
- need
  - towers
  - anchors
- don’t want movement

Cable Loads
- straight line between forces
- with one force
  - concurrent
  - symmetric

Cable Loads
- shape directly related to the distributed load
Cable-Stayed Structures

- diagonal cables support horizontal spans
- typically symmetrical
- Patcenter, Rogers 1986

Patcenter, Rogers 1986

- column free space
- roof suspended
- solid steel ties
- steel frame supports masts

Patcenter, Rogers 1986

- dashes – cables pulling

Moments

- forces have the tendency to make a body rotate about an axis

- same translation but different rotation
Moments

- defined by magnitude and direction
- units: \( N \cdot m, k \cdot ft \)
- direction:
  - + ccw (right hand rule)
  - - cw
- value found from \( F \) and \( \perp \) distance
  \[ M = F \cdot d \]
- \( d \) also called “lever” or “moment” arm

Moments

- a force acting at a different point causes a different moment:

\[ \neq \]

Moments

- with same \( F \):
  \[ M_A = F \cdot d_1 < M_A = F \cdot d_2 \] (bigger)
Moments

- additive with sign convention
- can still move the force along the line of action

Moments

- Varignon’s Theorem
  - resolve a force into components at a point and finding perpendicular distances
  - calculate sum of moments
  - equivalent to original moment
- makes life easier!
  - geometry
  - when component runs through point, \( d = 0 \)

Moments of a Force

- moments of a force
  - introduced in Physics as “Torque Acting on a Particle”
  - and used to satisfy rotational equilibrium

Physics and Moments of a Force

- my Physics book:
**Moment Couples**

- **2 forces**
  - same size
  - opposite direction
  - distance d apart
  - cw or ccw

\[ M = F \cdot d \]

- not dependant on point of application

\[ M = F \cdot d_1 - F \cdot d_2 \]

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**Moment Couples**

- equivalent couples
  - same magnitude and direction
  - F & d may be different

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**Moment Couples**

- added just like moments caused by one force
- can replace two couples with a single couple

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**Moment Couples**

- moment couples in structures
**Equivalent Force Systems**

- two forces at a point is equivalent to the resultant at a point
- resultant is equivalent to two components at a point
- resultant of equal & opposite forces at a point is zero
- put equal & opposite forces at a point (sum to 0)
- transmission of a force along action line

**Force-Moment Systems**

- single force causing a moment can be replaced by the same force at a different point by providing the moment that force caused
- moments are shown as arched arrows

**Parallel Force Systems**

- forces are in the same direction
- can find resultant force
- need to find location for equivalent moments