Mechanics of Materials

- **MECHANICS**
- **MATERIALS**

**Knowledge Required**

- material properties
- member cross sections
- ability of a material to resist breaking
- structural elements that resist excessive
  - deflection
  - deformation

**Mechanics of Materials**

- external loads and their effect on deformable bodies
- use it to answer question if structure meets requirements of
  - stability and equilibrium
  - strength and stiffness
- other principle building requirements
  - economy, functionality and aesthetics
Problem Solving

1. STATICS:
   - equilibrium of external forces, internal forces, stresses
2. GEOMETRY:
   - cross section properties, deformations and conditions of geometric fit, strains
3. MATERIAL PROPERTIES:
   - stress-strain relationship for each material obtained from testing

Stress

- stress is a term for the intensity of a force, like a pressure
- internal or applied
- force per unit area

\[
\text{stress} = f = \frac{P}{A}
\]

Design

- materials have a critical stress value where they could break or yield
  - ultimate stress
  - yield stress
  - compressive stress
  - fatigue strength
  - (creep & temperature)

Design (cont)

- we’d like \( f_{\text{actual}} \ll F_{\text{allowable}} \)
- stress distribution may vary: average
- uniform distribution exists IF the member is loaded axially (concentric)
Scale Effect

- **model scale**
  - material weights by volume, small section areas
- **structural scale**
  - much more material weight, bigger section areas
- scale for strength is not proportional:
  \[
  \frac{\gamma L^3}{L^2} = \gamma L
  \]

Normal Stress (direct)

- **normal stress is normal to the cross section**
  - stressed area is perpendicular to the load

\[
f_{t or c} = \frac{P}{A}
\]

Shear Stress

- stress parallel to a surface

\[
f_v = \frac{P}{A} = \frac{P}{td}
\]

Bearing Stress

- stress on a surface by contact in compression

\[
f_p = \frac{P}{A} = \frac{P}{td}
\]
**Bending Stress**

- normal stress caused by bending

\[ f_b = \frac{Mc}{I} = \frac{M}{S} \]

**Torsional Stress**

- shear stress caused by twisting

\[ f_v = \frac{T\rho}{J} \]

**Structures and Shear**

- what structural elements see shear?
  - beams
  - bolts
  - splices
  - slabs
  - footings
  - walls
    - wind
    - seismic loads

**Bolts**

- connected members in tension cause shear stress
- connected members in compression cause bearing stress
**Single Shear**
- seen when 2 members are connected

\[ f_v = \frac{P}{A} = \frac{P}{\pi \frac{d^2}{4}} \]

**Double Shear**
- seen when 3 members are connected
- two areas

\[ f_v = \frac{P}{2A} = \frac{P}{2A} = \frac{P}{\pi \frac{d^2}{4}} \]

**Bolt Bearing Stress**
- compression & contact
- projected area

\[ f_p = \frac{P}{A_{projected}} = \frac{P}{td} \]

**Strain**
- materials deform
- axially loaded materials change length
- bending materials deflect

\[ \text{strain} = \varepsilon = \frac{\Delta L}{L} \]
Shearing Strain

- deformations with shear
- parallelogram
- change in angles
- stress: $\tau$
- strain: $\gamma$
  - $\gamma = \frac{\delta_s}{L} = \tan \phi \equiv \phi$
  - unitless (radians)

Load and Deformation

- for stress, need $P$ & $A$
- for strain, need $\delta$ & $L$
  - how?
  - TEST with load and measure
  - plot $P/A$ vs. $\varepsilon$

Material Behavior

- every material has its own response
  - 10,000 psi
  - $L = 10$ in
  - Douglas Fir vs. steel?

Figure 5.20 Stress-strain diagram for various materials.
Behavior Types

- ductile - “necking”
- true stress 
  \[ f = \frac{P}{A} \]
- engineering stress 
  - (simplified) 
  \[ f = \frac{P}{A_0} \]

Behavior Types

- brittle
- semi-brittle

Stress to Strain

- important to us in \( f - \varepsilon \) diagrams:
  - straight section
  - LINEAR-ELASTIC
  - recovers shape (no permanent deformation)

Hooke’s Law

- straight line has constant slope
- Hooke’s Law 
  \[ f = E \cdot \varepsilon \]
- \( E \)
  - Modulus of elasticity
  - Young’s modulus
  - units just like stress
Stiffness

- ability to resist strain

- steels
  - same $E$
  - different yield points
  - different ultimate strength

Isotropy & Anisotropy

- **ISOTROPIC**
  - materials with $E$ same at any direction of loading
  - ex. steel

- **ANISOTROPIC**
  - materials with different $E$ at any direction of loading
  - ex. wood is orthotropic

Elastic, Plastic, Fatigue

- elastic springs back

- plastic has permanent deformation

- fatigue caused by reversed loading cycles

Plastic Behavior

- ductile
Lateral Strain

- or “what happens to the cross section with axial stress”
  \[ \varepsilon_x = \frac{f_x}{E} \]
  \[ f_y = f_z = 0 \]
- strain in lateral direction
  - negative
  - equal for isometric materials
  \[ \varepsilon_y = \varepsilon_z \]

Poisson’s Ratio

- constant relationship between longitudinal strain and lateral strain
  \[ \mu = -\frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x} \]
  \[ \varepsilon_y = \varepsilon_z = -\frac{\mu f_x}{E} \]
- sign!
  \[ 0 < \mu < 0.5 \]

Calculating Strain

- from Hooke’s law
  \[ f = E \cdot \varepsilon \]
- substitute
  \[ \frac{P}{A} = E \cdot \frac{\delta}{L} \]
- get ⇒ \[ \delta = \frac{PL}{AE} \]

Orthotropic Materials

- non-isometric
- directional values of \( E \) and \( \mu \)
- ex:
  - plywood
  - laminates
  - polymer composites
Stress Concentrations

• why we use $f_{ave}$
• increase in stress at changes in geometry
  – sharp notches
  – holes
  – corners

Maximum Stresses

• if we need to know where max $f$ and $f_v$ happen:

$$\theta = 0^\circ \rightarrow \cos \theta = 1 \quad f_{max} = \frac{P}{A_o}$$

$$\theta = 45^\circ \rightarrow \cos \theta = \sin \theta = \sqrt{0.5}$$

$$f_{v-max} = \frac{P}{2A_o} = \frac{f_{max}}{2}$$

Maximum Stresses

• physical movement
  – axially (same or zero)
  – rotations from axial changes

Deformation Relationships

• $\delta = \frac{PL}{AE}$ relates $\delta$ to $P$
Deformations from Temperature

- atomic chemistry reacts to changes in energy
- solid materials
  - can contract with decrease in temperature
  - can expand with increase in temperature
- linear change can be measured per degree

Thermal Deformation

- $\alpha$ - the rate of strain per degree
- UNITS: $/\degree F$, $/\degree C$
- length change: $\delta_T = \alpha(\Delta T)L$
- thermal strain: $\varepsilon_T = \alpha(\Delta T)$
  - no stress when movement allowed

Coefficients of Thermal Expansion

<table>
<thead>
<tr>
<th>Material</th>
<th>Coefficients ($\alpha$) [in./in./$\degree F$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood</td>
<td>$3.0 \times 10^{-6}$</td>
</tr>
<tr>
<td>Glass</td>
<td>$4.4 \times 10^{-6}$</td>
</tr>
<tr>
<td>Concrete</td>
<td>$5.5 \times 10^{-6}$</td>
</tr>
<tr>
<td>Cast Iron</td>
<td>$5.9 \times 10^{-6}$</td>
</tr>
<tr>
<td>Steel</td>
<td>$6.5 \times 10^{-6}$</td>
</tr>
<tr>
<td>Wrought Iron</td>
<td>$6.7 \times 10^{-6}$</td>
</tr>
<tr>
<td>Copper</td>
<td>$9.3 \times 10^{-6}$</td>
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<tr>
<td>Bronze</td>
<td>$10.1 \times 10^{-6}$</td>
</tr>
<tr>
<td>Brass</td>
<td>$10.4 \times 10^{-6}$</td>
</tr>
<tr>
<td>Aluminum</td>
<td>$12.8 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

Stresses and Thermal Strains

- if thermal movement is restrained stresses are induced
  1. bar pushes on supports
  2. support pushes back
  3. reaction causes internal stress $f = \frac{P}{A} = \frac{\delta E}{L}$
Superposition Method

- can remove a support to make it look determinant
- replace the support with a reaction
- enforce the geometry constraint

\[
\delta_p = -\frac{PL}{AE} \\
\delta_T = \alpha(\Delta T)L
\]

Factor of Safety

- accommodate uncertainty with a safety factor:
  \[
  \text{allowable load} = \frac{\text{ultimate load}}{F.S}
  \]

- with linear relation between load and stress:
  \[
  F.S = \frac{\text{ultimate load}}{\text{allowable load}} = \frac{\text{ultimate stress}}{\text{allowable stress}}
  \]

Design of Members

- beyond allowable stress...
- materials aren't uniform 100% of the time
  - ultimate strength or capacity to failure may be different and some strengths hard to test for
- RISK & UNCERTAINTY

\[
f_u = \frac{P_u}{A}
\]
Load and Resistance Factor Design

• loads on structures are
  – not constant
  – can be more influential on failure
  – happen more or less often
  – UNCERTAINTY

\[ R_u = \gamma_D R_D + \gamma_L R_L \leq \phi R_n \]

\( \phi \) - resistance factor
\( \gamma \) - load factor for (D)ead & (L)ive load