ARCH 331: Practice Quiz 5

Note: No aids are allowed for part 1. One side of a letter sized paper with notes is allowed during part 2, along with a silent, non-programmable calculator. There are reference charts for part 2, shown on pages 2-6.

Clearly show your work and answer.

Part 1) Worth 5 points (conceptual questions)

Part 2) Worth 45 points

(NOTE: The loading type [ex, live, dead, wind...] and sizes can and will be changed for the quiz with respect to the beam diagrams and formula provided. The support condition, section, and bracing for the column can and will be changed.)

A wide flange beam of A992 steel (F_y = 50 ksi, E = 29 x 10^3 ksi) is needed to span 32 ft and support uniformly distributed loads of 980 lb/ft of dead load (from materials), the self weight, and 1150 lb/ft of live load over a length of 11 feet as shown. The beam is simply supported with a maximum unbraced length of 15.5 ft.

a) Select the most economical beam adequate for flexural strength using LRFD design and the chart provided (including self weight). Assume that the dead load determines the location of the maximum moment and superimpose the live load moment there.

b) Determine the minimum moment of inertia required such that the dead load deflection does not exceed 1.25 inches assuming a self weight of 60 lb/ft. [or live load deflection– using
\[ \Delta_{max} = \frac{wL^4}{152EI} \] because there is no equation – does not exceed 0.8 in; or total deflection
assuming that the dead load determines the location of the maximum deflection – using
\[ \Delta_t = 5wx(L^4 - 3lx^2 + 2x^3)/(92EI) \] because there is no equation for the partial distributed load – does not exceed 1.75 in.]

A W 250 x 49 metric column is 5.75 m tall of A36 steel (F_y = 250 MPa, E = 200 x 10^3 MPa). The base is fixed and the top is pinned in the weak axis, while the strong axis is considered pinned at the top and bottom (no picture and approximated conditions). The section properties are:
- A = 6260 mm^2, I_x = 70.7 x 10^6 mm^4, r_x = 106 mm, I_y = 15.2 x 10^6 mm^4, r_y = 49.3 mm

A) If the column is to support 200 kN of dead load and 600 kN of live load, is it adequate for design using LRFD?

Answers – Not provided on actual quiz!

a) M_u = 279 k-ft, use W21x55 (M_u < 319 k-ft)
b) I_{req, dead only} = 677 in^4 (dead only) [I_{req, dead only} = 591 in^4; I_{req, total} = 751 in^4]
c) \phi P_a = 886 kN : . Not OK (weak axis governs because \phi P_{a, strong} = 1199 kN)
REFERENCE CHARTS FOR QUIZ 5

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD

\[ R = \frac{w}{2} \]

\[ M_{\text{max.}} (\text{at center}) = \frac{w}{8} \]

\[ \Delta x = \frac{w}{24EI} (\frac{1}{8}x^2 - \frac{1}{4}x^4) \]

2. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO ONE END

\[ W = \frac{w}{2} \]

\[ R_1 = \frac{3W}{2} \]

\[ V_x = \frac{W}{3} - \frac{Wx^2}{2} \]

\[ M_{\text{max.}} (\text{at } x = \frac{l}{\sqrt{3}}) = \frac{2Wl}{3} \]

\[ \Delta x = \frac{Wx}{180EI} \] \[ (3x^4 - 10x^2 + 7l^4) \]

3. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO CENTER

\[ W = \frac{w}{2} \]

\[ V_x (\text{when } x < \frac{l}{2}) = \frac{W}{2} \] \[ (\frac{1}{2}x^2 - 4x^4) \]

\[ M_{\text{max.}} (\text{at center}) = \frac{Wl}{6} \]

\[ M_{\text{max.}} (\text{at center}) = \frac{Wx}{2} \] \[ (\frac{1}{2}x^2) \]

\[ \Delta x (\text{when } x < \frac{l}{2}) = \frac{Wx}{480EI} \] \[ (\frac{1}{8}x^2 - \frac{1}{4}x^4) \]

4. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED

\[ R_1 = V_1 (\text{max. when } a < c) = \frac{w_1}{2l} (2c + b) = \frac{w_1}{2} - \frac{w_2}{2l} \]

\[ R_2 = V_2 (\text{max. when } a > c) = \frac{w_2}{2l} (2a + b) = \frac{w_2}{2} \]

\[ V_x (\text{when } x > a \text{ and } x < (a + b)) = V_1 - w(x - a) \]

\[ M_{\text{max.}} (\text{at } x = a + \frac{R_1}{w}) = R_2 \left( a + \frac{R_1}{2w} \right) \]

\[ M_{\text{max.}} (\text{at } x = a + \frac{R_1}{w}) = R_1x - \frac{w_1}{2} \]

\[ M_{\text{max.}} (\text{at } x > (a + b)) = R_2 (l - x) \]

5. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT ONE END

\[ R_1 = V_1 \text{ max.} = \frac{w_1}{2l} (2l - a) = \frac{w_1}{2} - \frac{w_2}{2l} \]

\[ R_2 = V_2 \text{ max.} = \frac{w_2}{2l} = \frac{w_1}{2l} \]

\[ V_x (\text{when } x < a) = V_1 - w(x - a) \]

\[ M_{\text{max.}} (\text{at } x = a) = R_2 \left( a + \frac{R_1}{2w} \right) \]

\[ M_{\text{max.}} (\text{at } x > a) = R_2 (l - x) \]

\[ \Delta x (\text{when } x < a) = \frac{w_1 x^2}{24EI} \] \[ (a^2 - 2ax - 2ax \frac{1}{2} + 2x^2 - a^2) \]

6. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT EACH END

\[ R_1 = V_1 \text{ max.} = \frac{w_1}{2l} (2l - a) + \frac{w_2}{2l} \]

\[ R_2 = V_2 \text{ max.} = \frac{w_2}{2l} \]

\[ V_x (\text{when } x < a) = V_1 - w(x - a) \]

\[ M_{\text{max.}} (\text{at } x = a) = R_2 \left( a + \frac{R_1}{2w} \right) \]

\[ M_{\text{max.}} (\text{at } x > a) = R_2 (l - x) \]

\[ \Delta x (\text{when } x < a) = \frac{w_1 x^2}{24EI} \] \[ (a^2 - 2ax - 2ax \frac{1}{2} + 2x^2 - a^2) \]
REFERENCE CHARTS FOR QUIZ 5

13. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—CONCENTRATED LOAD AT CENTER

Total Equiv. Uniform Load $= \frac{3P}{2}$

$R_1 = V_1 = \frac{5P}{16}$

$R_2 = V_2 = 11P/16$

$M_{\text{max. at fixed end}} = \frac{3P}{16}$

$M_1 = \text{at point of load} = \frac{3P}{2}$

$M_x (\text{when } x < \frac{1}{2}) = 5P_x/16$

$M_x (\text{when } x > \frac{1}{2}) = \frac{P}{2} \left(\frac{1}{2} - \frac{11x}{16}\right)$

$\Delta_{\text{max. at } x = l \sqrt{\frac{1}{5} = 0.4472}} = \frac{P_{/}}{48EI} \sqrt{\frac{1}{5}} = \frac{P_{/}}{48EI} \sqrt{\frac{1}{5}}$

$\Delta_x (\text{at point of load}) = \frac{7P}{96EI}$

$\Delta_x (\text{when } x < \frac{1}{2}) = \frac{P_x}{96EI} (3x^2 - 5x^4)$

$\Delta_x (\text{when } x > \frac{1}{2}) = \frac{P}{96EI} (x - l)^2 (11x - 2l)$

14. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—CONCENTRATED LOAD AT ANY POINT

$R_1 = V_1 = \frac{Pb^2}{2^2} (a+2l) = P - R_2$

$R_2 = V_2 = \frac{Pa}{2^2} (3l^2 - a^2) = P - R_1$

$M_1 (\text{at point of load}) = R_1 a$

$M_2 (\text{at fixed end}) = \frac{Pab}{2^2} (a + l)$

$M_x (\text{when } x < a) = R_1 x$

$M_x (\text{when } x > a) = R_1 x - P (x - a)$

$\Delta_{\text{max. when } x < \frac{a}{2}} = \frac{P_{/}}{3EI} (3l^2 - a^2)$

$\Delta_{\text{max. when } x > \frac{a}{2}} = \frac{P_{/}}{6EI} \sqrt{\frac{1}{3}} (a^2 + l - a)$

$\Delta_x (\text{at point of load}) = \frac{P_{/a} b}{12EI} (3l^2 + a)$

$\Delta_x (\text{when } x < a) = \frac{P_{/a} b}{12EI} (3l^2 - 2x^2 - ax^2)$

$\Delta_x (\text{when } x > a) = \frac{P_{/a} b}{12EI} (l - x)^2 (3l - x^2 - 2a^2)$

15. BEAM FIXED AT BOTH ENDS—UNIFORMLY DISTRIBUTED LOADS

Total Equiv. Uniform Load $= \frac{2wL}{3}$

$R = V \cdot \frac{wL}{2}$

$V_x = \frac{w(\frac{L}{2} - x)}{2}$

$M_{\text{max. at ends}} = \frac{wL}{12}$

$M_1 (\text{at center}) = \frac{wL}{24}$

$M_x (\text{at center}) = \frac{w(6I - l^2 - 6x^2)}{24l^4}$

$\Delta_{\text{max. at center}} = \frac{38wE}{12EI} E$

$\Delta_x (\text{when } x < \frac{L}{2}) = \frac{wx^2}{24EI} (l - x)^2$

16. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT CENTER

Total Equiv. Uniform Load $= P$

$R = V \cdot \frac{P}{2}$

$M_{\text{max. at center and ends}} = \frac{P}{3}$

$M_x (\text{when } x < \frac{1}{2}) = \frac{P}{6} (4x - l)$

$M_x (\text{at center}) = \frac{192EI}{P}$

$\Delta_{\text{max. at center}} = \frac{P_x}{96EI} (3l - 4x)$

17. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT ANY POINT

$R_1 = V_1 (\text{max. when } a < b) = \frac{Pb^2}{l^2} (3a + b) = P - R_2$

$R_2 = V_2 (\text{max. when } a > b) = \frac{Pa^2}{l^2} (a - 3b) = P - R_1$

$M_1 (\text{max. when } a < b) = \frac{Pab}{l^2}$

$M_2 (\text{max. when } a > b) = \frac{2Pab}{l^2}$

$M_x (\text{at point of load}) = \frac{R_1 x - Pab}{l^2}$

$M_x (\text{when } x < a) = \frac{R_1 x - Pab}{l^2}$

$\Delta_{\text{max. when } a > b = \frac{2al}{3EI} (3a + b)^{\frac{1}{2}}}$

$\Delta_{\text{max. when } a < b = \frac{2al}{3EI} (3a + b)^{\frac{1}{2}}}$

$\Delta_x (\text{at point of load}) = \frac{Pab^2}{6EI l^2}$

$\Delta_x (\text{when } x < a) = \frac{Pab^2}{6EI l^2}$
REFERENCE CHARTS FOR QUIZ 5
**REFERENCE CHARTS FOR QUIZ 5**

Available Critical Stress, $\phi F_{cr}$, for Compression Members, MPa \((F_y = 250\text{ MPa and } \phi = 0.90)\)

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<td>120</td>
<td>104.7</td>
<td>160</td>
<td>60.8</td>
<td>200</td>
<td>38.9</td>
</tr>
</tbody>
</table>
## Reference Charts for Quiz 5

### Buckled Shape of Column
- Shown by dashed line

<table>
<thead>
<tr>
<th></th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theoretical $K$ value</td>
<td>0.5</td>
<td>0.7</td>
<td>1.0</td>
<td>1.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Recommended design values when ideal conditions are approximated</td>
<td>0.65</td>
<td>0.80</td>
<td>1.0</td>
<td>1.2</td>
<td>2.10</td>
<td>2.0</td>
</tr>
</tbody>
</table>

### End Conditions Code
- Rotation fixed, Translation fixed
- Rotation free, Translation fixed
- Rotation fixed, Translation free
- Rotation free, Translation free