lecture twenty two

concrete construction:
flat spanning systems, columns & frames

http://nisee.berkeley.edu/godden
Reinforced Concrete Design

- economical & common
- resist lateral loads
Reinforced Concrete Design

• **flat plate**
  - 5”-10” thick
  - simple formwork
  - lower story heights

• **flat slab**
  - same as plate
  - 2 ¼”–8” drop panels
Reinforced Concrete Design

• **beam supported**
  - slab depth ~ \( L/20 \)
  - 8”–60” deep

• **one-way joists**
  - 3”–5” slab
  - 8”–20” stems
  - 5”-7” webs
Reinforced Concrete Design

• two-way joist
  – “waffle slab”
  – 3”-5” slab
  – 8”-24” stems
  – 6”-8” webs

• beam supported slab
  – 5”-10” slabs
  – taller story heights
Reinforced Concrete Design

- simplified frame analysis
  - strips, like continuous beams
- moments require flexural reinforcement
  - top & bottom
  - both directions of slab
  - continuous, bent or discontinuous
Reinforced Concrete Design

- one-way slabs (wide beam design)
  - approximate analysis for moment & shear coefficients
  - two or more spans
  - ~ same lengths
  - $w_u$ from combos
  - uniform loads with $L/D \leq 3$
  - $l_n$ is clear span (+M) or average of adjacent clear spans (-M)
Reinforced Concrete Design

Figure 2-3 Positive Moments—All Cases

Figure 2-4 Negative Moments—Beams and Slabs
Reinforced Concrete Design

- **two-way slabs - Direct Design Method**
  - 3 or more spans each way
  - uniform loads with $L/D \leq 3$
  - rectangular panels with long/short span $\leq 2$
  - successive spans can’t differ $> \frac{\text{longer}}{3}$
  - column offset no more than 10% span
## Reinforced Concrete Design

### Table 4-6 Two-Way Beam-Supported Slab

<table>
<thead>
<tr>
<th>Span ratio</th>
<th>Slab Moments</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Exterior</td>
<td>Positive</td>
<td>First Interior</td>
<td>Positive</td>
<td>Interior</td>
</tr>
<tr>
<td>0.5</td>
<td>Total Moment</td>
<td>0.16 $M_o$</td>
<td>0.57 $M_o$</td>
<td>0.70 $M_o$</td>
<td>0.35 $M_o$</td>
<td>0.65 $M_o$</td>
</tr>
<tr>
<td></td>
<td>Column Strip</td>
<td>0.12 $M_o$</td>
<td>0.43 $M_o$</td>
<td>0.54 $M_o$</td>
<td>0.27 $M_o$</td>
<td>0.50 $M_o$</td>
</tr>
<tr>
<td></td>
<td>Beam Slab</td>
<td>0.02 $M_o$</td>
<td>0.08 $M_o$</td>
<td>0.09 $M_o$</td>
<td>0.05 $M_o$</td>
<td>0.09 $M_o$</td>
</tr>
<tr>
<td></td>
<td>Middle Strip</td>
<td>0.02 $M_o$</td>
<td>0.06 $M_o$</td>
<td>0.07 $M_o$</td>
<td>0.03 $M_o$</td>
<td>0.06 $M_o$</td>
</tr>
<tr>
<td>1.0</td>
<td>Total Moment</td>
<td>0.10 $M_o$</td>
<td>0.37 $M_o$</td>
<td>0.45 $M_o$</td>
<td>0.22 $M_o$</td>
<td>0.42 $M_o$</td>
</tr>
<tr>
<td></td>
<td>Column Strip</td>
<td>0.02 $M_o$</td>
<td>0.06 $M_o$</td>
<td>0.08 $M_o$</td>
<td>0.04 $M_o$</td>
<td>0.07 $M_o$</td>
</tr>
<tr>
<td></td>
<td>Beam Slab</td>
<td>0.04 $M_o$</td>
<td>0.14 $M_o$</td>
<td>0.17 $M_o$</td>
<td>0.09 $M_o$</td>
<td>0.16 $M_o$</td>
</tr>
<tr>
<td>2.0</td>
<td>Total Moment</td>
<td>0.06 $M_o$</td>
<td>0.22 $M_o$</td>
<td>0.27 $M_o$</td>
<td>0.14 $M_o$</td>
<td>0.25 $M_o$</td>
</tr>
<tr>
<td></td>
<td>Column Strip</td>
<td>0.01 $M_o$</td>
<td>0.04 $M_o$</td>
<td>0.05 $M_o$</td>
<td>0.02 $M_o$</td>
<td>0.04 $M_o$</td>
</tr>
<tr>
<td></td>
<td>Beam Slab</td>
<td>0.09 $M_o$</td>
<td>0.31 $M_o$</td>
<td>0.38 $M_o$</td>
<td>0.19 $M_o$</td>
<td>0.36 $M_o$</td>
</tr>
</tbody>
</table>

**Notes:**

1. Beams and slab satisfy stiffness criteria: $\alpha_1 l_2 / l_1 \geq 1.0$ and $\beta_1 \geq 2.5$.
2. Interpolate between values shown for different $l_2/l_1$ ratios.
3. All negative moments are at face of support.
4. Concentrated loads applied directly to beams must be accounted for separately.
Shear in Concrete

• at columns
• want to avoid stirrups
• can use shear studs or heads
Shear in Concrete

- critical section at $d/2$ from
  - column face, column capital or drop panel
Shear in Concrete

• at columns with waffle slabs
Openings in Slabs

- careful placement of holes
- shear strength reduced
- bending & deflection can increase
General Beam Design

- $f'_c$ & $f_y$ needed
- usually size just $b$ & $h$
  - even inches typical (forms)
  - similar joist to beam depth
  - $b:h$ of 1:1.5-1:2.5
  - $b_w$ & $b_f$ for $T$
  - to fit reinforcement + stirrups
- slab design, $t$
  - deflection control & shear

$$S = \frac{bh^2}{6}$$

Figure 14.5 Common shapes for beams.
General Beam Design (cont’d)

• **custom design:**
  - longitudinal steel
  - shear reinforcement
  - detailing
Space “Frame” Behavior

• handle uniformly distributed loads well

• bending moment
  – tension & compression “couple” with depth
  – member sizes can vary, but difficult
Space “Frame” Behavior

- shear at columns
- support conditions still important
  - point supports not optimal
- fabrication/construction can dominate design
Folded Plates

- increased bending stiffness with folding
- lateral buckling avoided

(a) roof plan

(b) roof plan

compression
dashed line tension
Folded Plates

- common for roofs
- edges need stiffening

[Images of architectural structures]
Folded Plates

- State Farm Center (Assembly Hall), University of Illinois
- Harrison & Abramovitz 1963
- Edge-supported dome spanning 400 feet wound with 614 miles of one-fifth inch steel wire
Concrete in Compression

- crushing
- vertical cracking
  - tension
- diagonal cracking
  - shear
- $f'_c$
Columns Reinforcement

- columns require
  - ties or spiral reinforcement to "confine" concrete (#3 bars minimum)
  - minimum amount of longitudinal steel (4 bars minimum)
Slenderness

- effective length in monolithic with respect to stiffness of joint: $\Psi \& k$
- not slender when

$$\frac{kL_u}{r} \leq 22$$

*not braced

Fixed
Effective Length (revisited)

- relative rotation

\[
\Psi = \frac{\sum EI}{\sum EI/l_c}/\frac{l_c}{l_b}
\]
Column Behavior

Figure 13.3.2  Spirally reinforced column behavior. (Courtesy of Portland Cement Association.)

Figure 13.3.3  Tied column behavior. (Courtesy of Portland Cement Association.)
Column Design

- $\phi_c = 0.65$ for ties, $\phi_c = 0.75$ for spirals
- $P_o$ - no bending
  \[ P_o = 0.85 f_c'(A_g - A_{st}) + f_y A_{st} \]
- $P_u \leq \phi_c P_n$
  - ties: $P_n = 0.8P_o$
  - spiral: $P_n = 0.85P_o$
- **nominal axial capacity:**
  - presumes steel yields
  - concrete at ultimate stress
Columns with Bending

- eccentric loads can cause moments
- moments can change shape and induce more deflection ($P-\Delta$)

![Diagram of columns with bending](image)

**Figure 10.6** Considerations for development of bending in steel columns; (a) bending induced by eccentric load, (b) bending transferred to column in a rigid frame, and (c) combined loading condition, separately producing axial compression and bending.
Columns with Bending

- for ultimate strength behavior, ultimate strains can’t be exceeded
  - concrete 0.003
  - steel \( \frac{f_y}{E_s} \)

- \( P \) reduces with \( M \)

Figure 13.6.1 Typical strength interaction diagram for axial compression and bending moment about one axis. Transition zone is where \( \epsilon_i \leq \epsilon_t \leq 0.005 \).
Columns with Bending

- need to consider combined stresses
- linear strain
- steel stress at or below $f_y$
- plot interaction diagram

Figure 5-3 Transition Stages on Interaction Diagram
Design Methods

- calculation intensive
  - handbook charts
  - computer programs
Design Considerations

- **bending at both ends**
  - $P - \Delta$ maximum

- **biaxial bending**

- **walls**
  - unit wide columns
  - “deep” beam shear

- **detailing**
  - shorter development lengths
  - dowels to footings