point equilibrium
and planar trusses
Equilibrium

• balanced
• steady
• resultant of forces on a particle is 0
Equilibrium on a Point

- **analytically**

\[
R_x = \sum F_x = 0
\]
\[
R_y = \sum F_y = 0
\]

- **Newton convinces us it will stay at rest**
Equilibrium on a Point

- **collinear force system**
  - *ex: cables*

\[ \sum F_{in-line} = 0 \]

\[
\begin{align*}
R_x &= \sum F_x = 0 \\
R_y &= \sum F_y = 0
\end{align*}
\]
Equilibrium on a Point

• concurrent force system
  – ex: cables

\[
R_x = \sum F_x = 0
\]

\[
R_y = \sum F_y = 0
\]
Free Body Diagram

- **FBD (sketch)**
- **tool to see all forces on a body or a point including**
  - external forces
  - weights
  - force reactions
  - internal forces
Free Body Diagram

- determine point
- FREE it from:
  - ground
  - supports & connections
- draw all external forces acting ON the body
  - reactions (supporting forces)
  - applied forces
  - gravity

Sign suspended from a strut and cable.

FBD of concurrent point B.
Free Body Diagram

- sketch FBD with relevant geometry
- resolve each force into components
  - known & unknown angles – name them
  - known & unknown forces – name them
- are any forces related to other forces?
- for the unknowns
- write only as many equilibrium equations as needed
- solve up to 2 equations
Free Body Diagram

- solve equations
  - most times 1 unknown easily solved
  - plug into other equation(s)

- common to have unknowns of
  - force magnitudes
  - force angles
Truss Structures

- ancient (?) wood
  - Romans 500 B.C.
- Renaissance revival
- 1800’s analysis
- efficient
Truss Structures

– analogous to cables and struts

(a) STABLE: pinned supports resist thrust

(b) UNSTABLE: substitution of roller support eliminates thrust resistance

(c) STABLE: wood strut resists thrust internally to form simple truss
Truss Structures

- comprised of straight members
- geometry with triangles is stable
- loads applied only at pin joints
Truss Structures

- **2 force members**
  - forces in line, equal and opposite
  - compression
  - tension

- **3 members connected by 3 joints**

- **2 more members need 1 more joint**
  \[ b = 2n - 3 \]
Truss Structures

- compression and tension
Truss Structures

- statically determinate
- indeterminate
- unstable

\( b = 21 \)
\[ n = 12 \quad 2(n) - 3 = 2(12) - 3 = 21 \]
(a) Determinate.

\( b = 16 \)
\[ n = 10 \quad b = 16 < 2(10) - 3 = 17 \]
(Too few members—square panel is unstable)

\( b = 18 \)
\[ n = 10 \quad b = 18 > 2(10) - 3 = 17 \]
(Too many members)

(b) Indeterminate.
Trusses

• common designs

(a) King post
Inverted king post

(b) Queen post
Inverted queen post

(c) Pitched Pratt truss

(d) Pitched Howe truss

(e) Constant forces in upper chords and no forces in diagonals (normally built with rigid joints if diagonals are omitted).

(f) Pitched Fink truss

(g) Parallel chord
Warren trusses

(h) Parallel chord
Pratt truss

(i) Parallel chord
Howe truss

(k) Parallel chord
crossed-diagonal truss
Trusses

• common designs

(i) Bowstring truss

(m) Lenticular truss

(w) “Scissors” truss

(x) Cantilevered truss (funicularly shaped)

(y) Northlight trusses

(z) Monitors with clerestories
Trusses

• uses
  – roofs & canopies
  – long spans
  – lateral bracing
Truss Connections

- “pins”
Sainsbury Center, Foster 1978

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Architectural Structures
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two pin-connection supports
(typical of all trusses)

see detail

third pin connection
at end trusses only
(makes truss and supporting columns behave as a rigid frame to minimize movement around end glazing)

prism (3-sided) roof trusses

tubular cross-bracing between columns

tubular steel prism columns are cantilevered from foundation (rigid base connection)
Truss Analysis

• visualize compression and tension from deformed shape
Truss Analysis

- **Method of Joints**
- **Graphical Methods**
- **Method of Sections**

- all rely on equilibrium
  - of bodies
  - internal equilibrium
Method of Joints

- isolate each joint
- enforce equilibrium in $F_x$ and $F_y$
- can find all forces

- long
- easy to mess up
Joint Cases

- *two bodies connected*

![Image showing joint cases with labels A, B, C, and equal forces or moments](image-url)
Joint Cases

- three bodies with two in line
Joint Cases

• crossed

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Lecture 3

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**Tools – Multiframe**

- *in classrooms and open access labs*

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*Images of software interfaces showing Multiframe tool.*
Tools – Multiframe

- **frame window**
  - define truss members
    - or pre-defined truss
  - select points, assign supports
  - select members, assign section & assign pin ends

- **load window**
  - select points, add point load
Tools – Multiframe

• to run analysis choose
  – Analyze menu
    • Linear

• plot
  – choose options

• results
  – choose options