

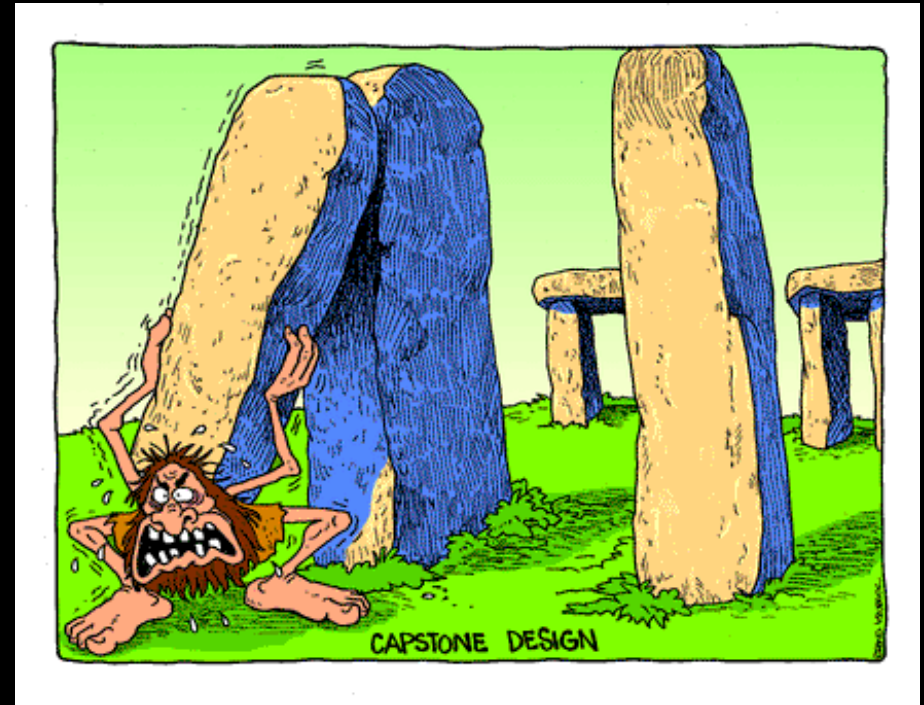
**ARCHITECTURAL STRUCTURES:
FORM, BEHAVIOR, AND DESIGN**

ARCH 331

DR. ANNE NICHOLS

SUMMER 2018

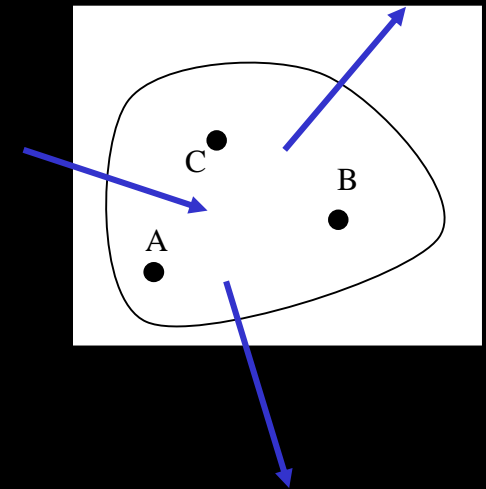
**lecture
four**



**rigid body
equilibrium**

Equilibrium

- *rigid body*
 - doesn't deform
 - coplanar force systems



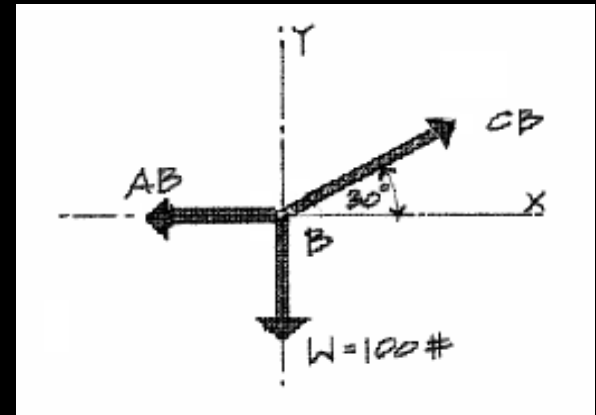
- *static:* $R_x = \sum F_x = 0$

$$R_y = \sum F_y = 0$$

$$M = \sum M = 0$$

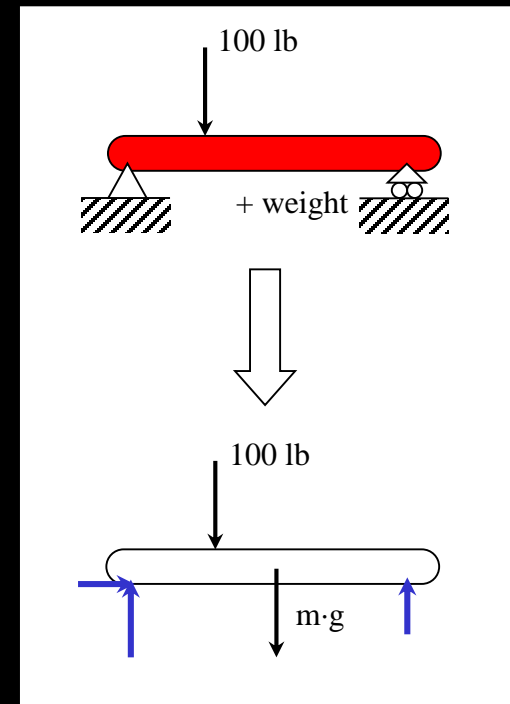
Free Body Diagram

- *FBD (sketch)*
- *tool to see all forces on a body or a point including*
 - *external forces*
 - *weights*
 - *force reactions*
 - *external moments*
 - *moment reactions*
 - *internal forces*



Free Body Diagram

- *determine body*
- *FREE it from:*
 - *ground*
 - *supports & connections*
- *draw all external forces acting ON the body*
 - *reactions*
 - *applied forces*
 - *gravity*



Free Body Diagram

- *sketch FBD with relevant geometry*
- *resolve each force into components*
 - *known & unknown angles – name them*
 - *known & unknown forces – name them*
 - *known & unknown moments – name them*
- *are any forces related to other forces?*
- *for the unknowns*
- *write only as many equilibrium equations as needed*
- *solve up to 3 equations*

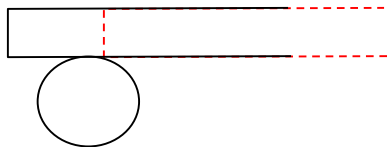
Free Body Diagram

- *solve equations*
 - *most times 1 unknown easily solved*
 - *plug into other equation(s)*

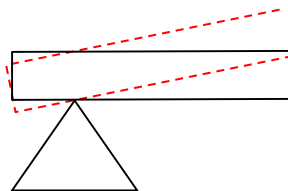
- *common to have unknowns of*
 - *force magnitudes*
 - *force angles*
 - *moment magnitudes*

Reactions on Rigid Bodies

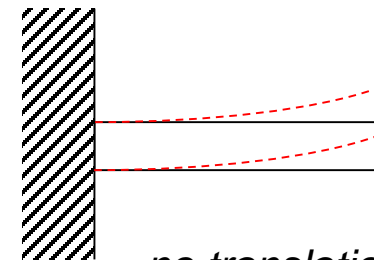
- *result of applying force*
- *unknown size*
- *connection or support type*
 - *known direction*
 - *related to motion prevented*



no vertical motion

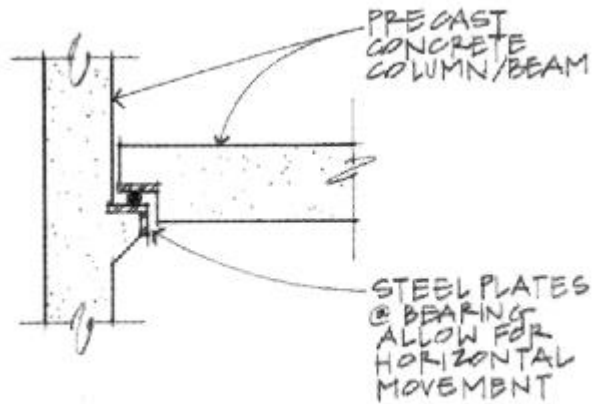


no translation

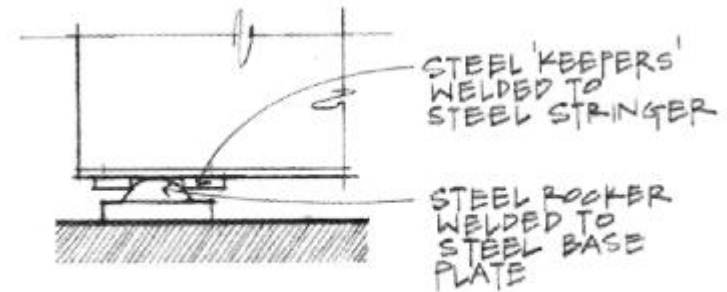


*no translation
no rotation*

Supports and Connections



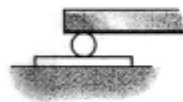
Roller



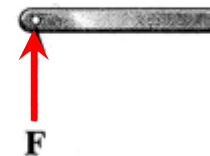
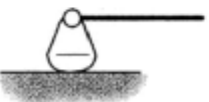
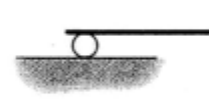
Rocker



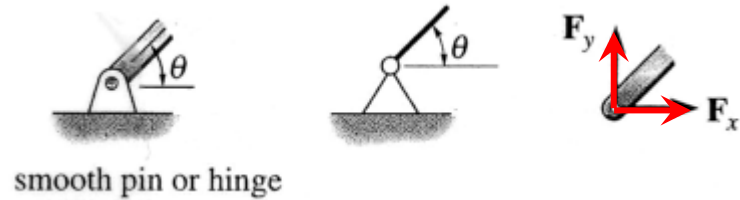
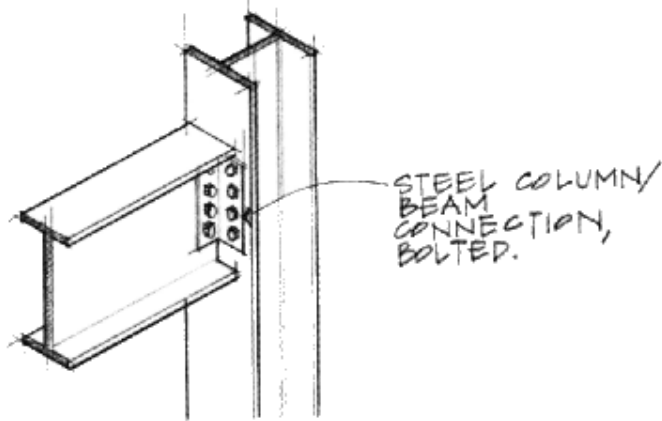
rollers



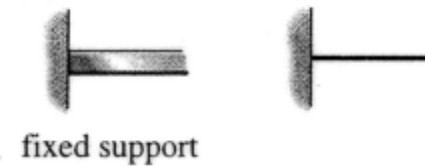
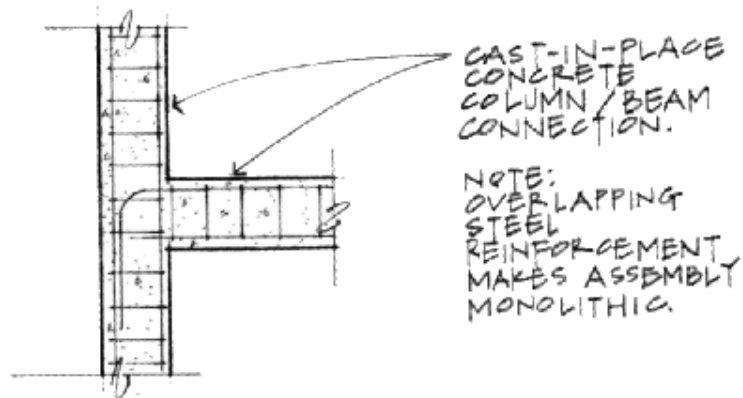
rocker



Supports and Connections



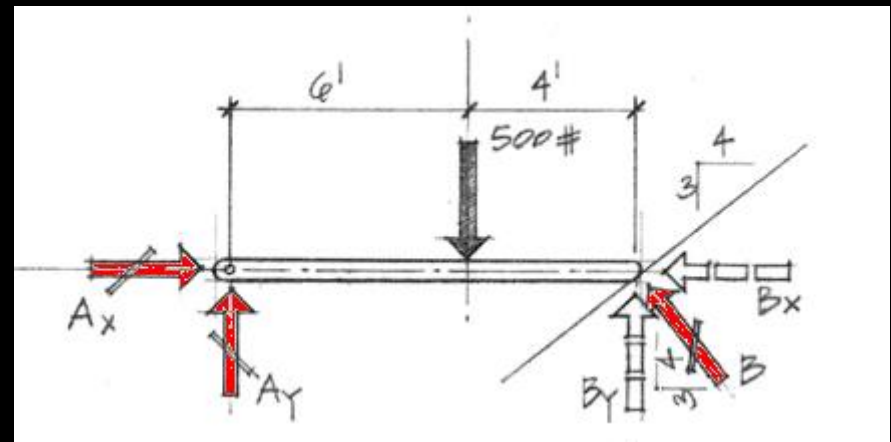
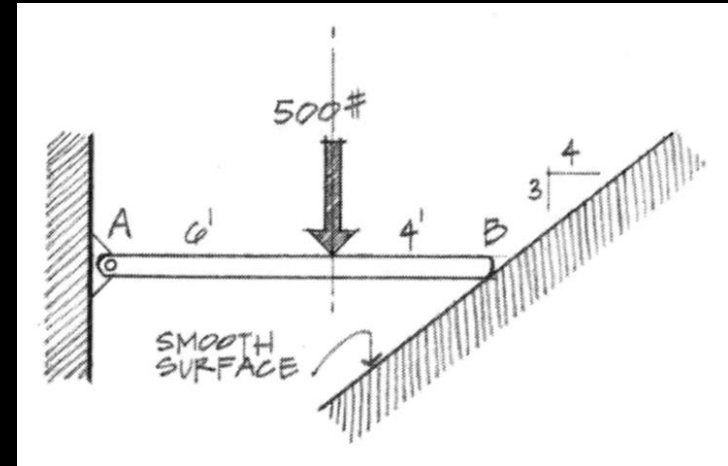
Pin



Fixed

FBD Example

- 500 lb known
- pin – A_x , A_y
- smooth surface –
B at 4:3
- 3 equations
- sum moments at
 - A?
 - B? (B_x)



Moment Equations

- *sum moments at intersection where the most forces intersect*
- *multiple moment equations may not be useful*
- *combos:*

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M_1 = 0$$

$$\sum F = 0$$

$$\sum M_1 = 0$$

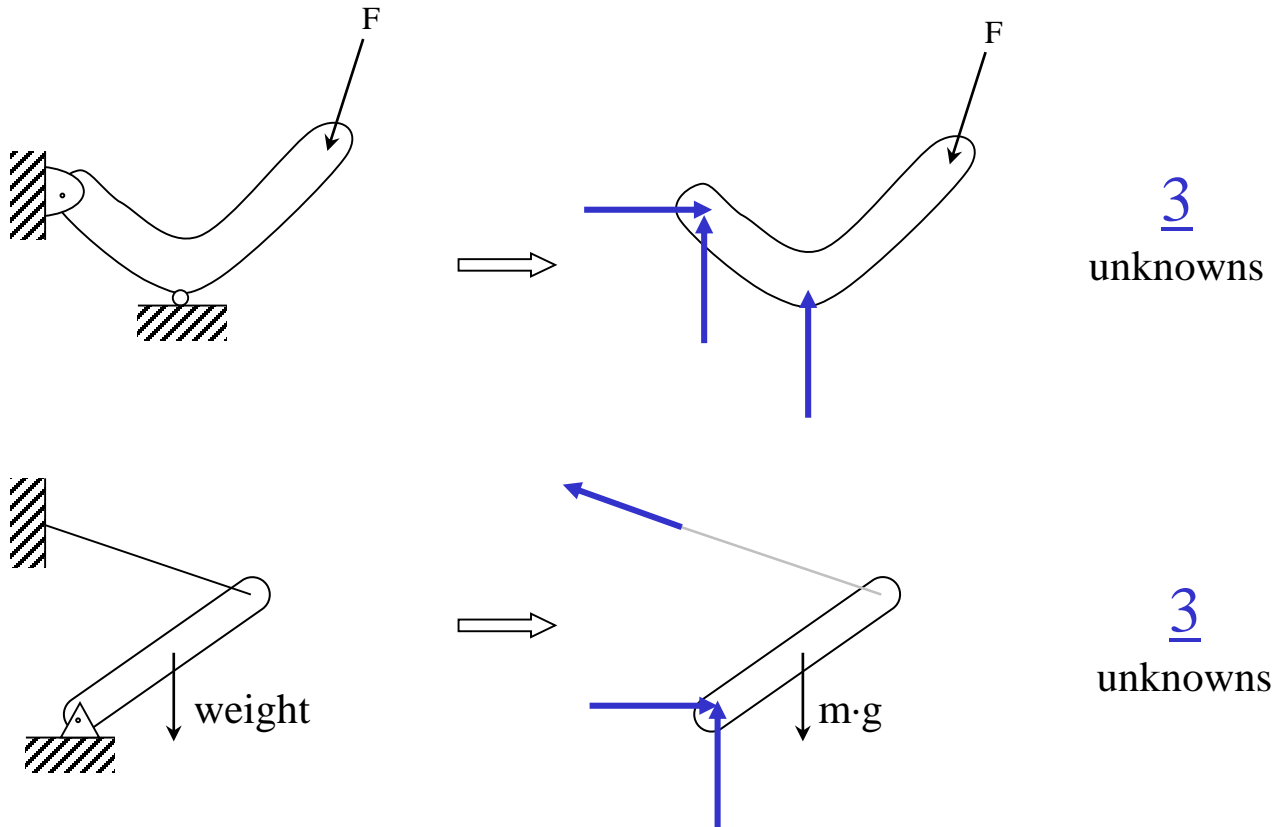
$$\sum M_2 = 0$$

$$\sum M_1 = 0$$

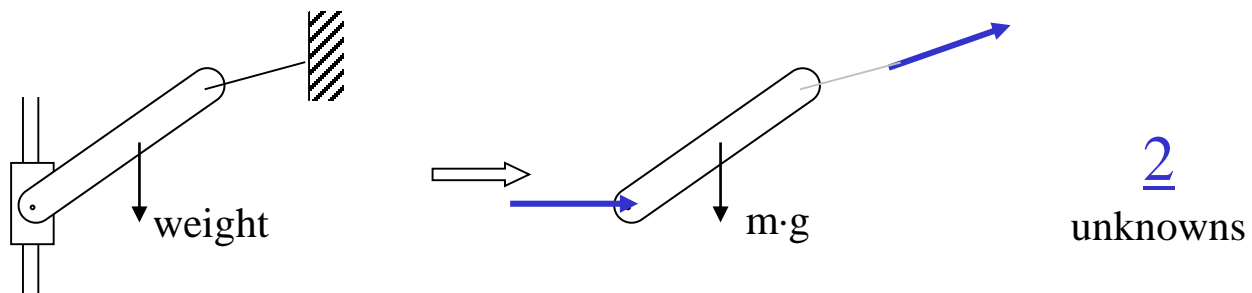
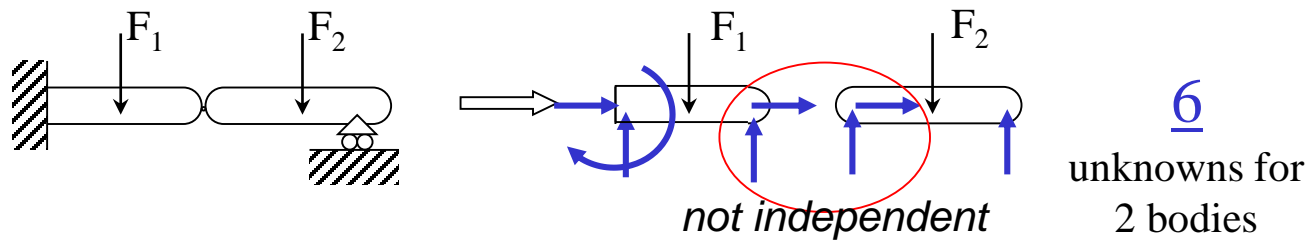
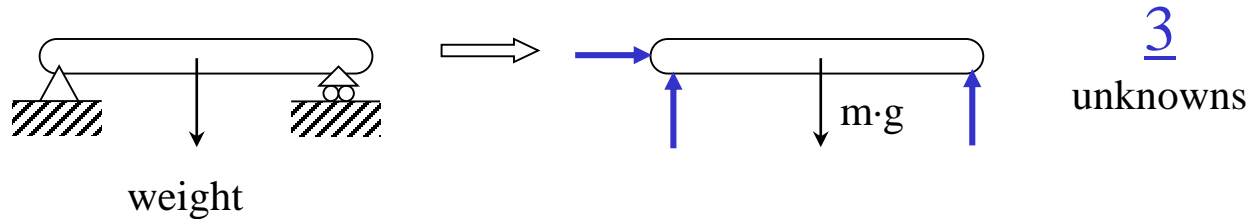
$$\sum M_2 = 0$$

$$\sum M_3 = 0$$

Recognizing Reactions

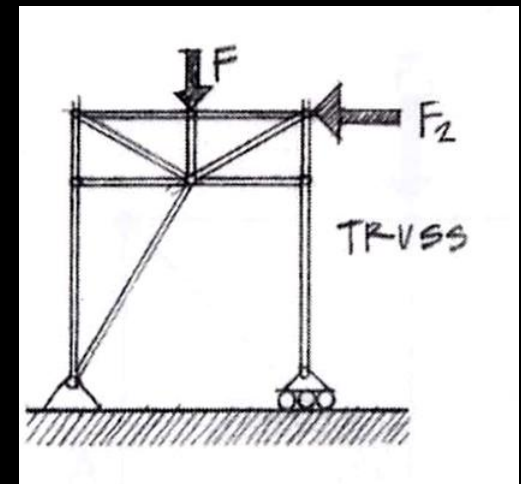
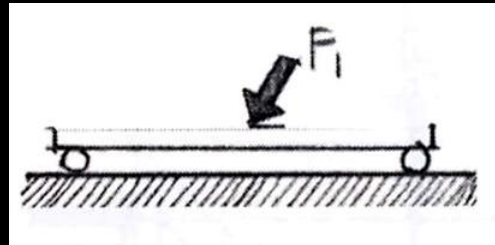
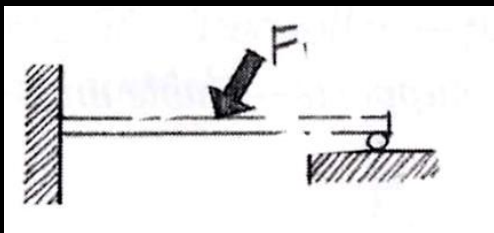


Recognizing Reactions



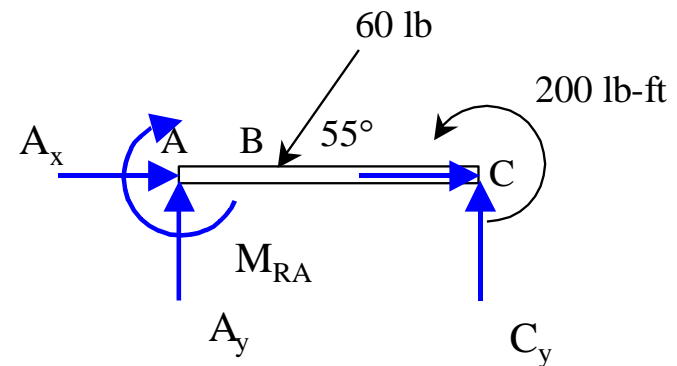
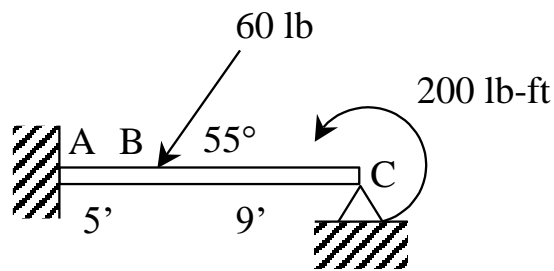
Constraints

- *completely constrained*
 - doesn't move
 - may not be statically determinate
- *improperly or partially constrained*
 - has \leq unknowns
 - can't solve

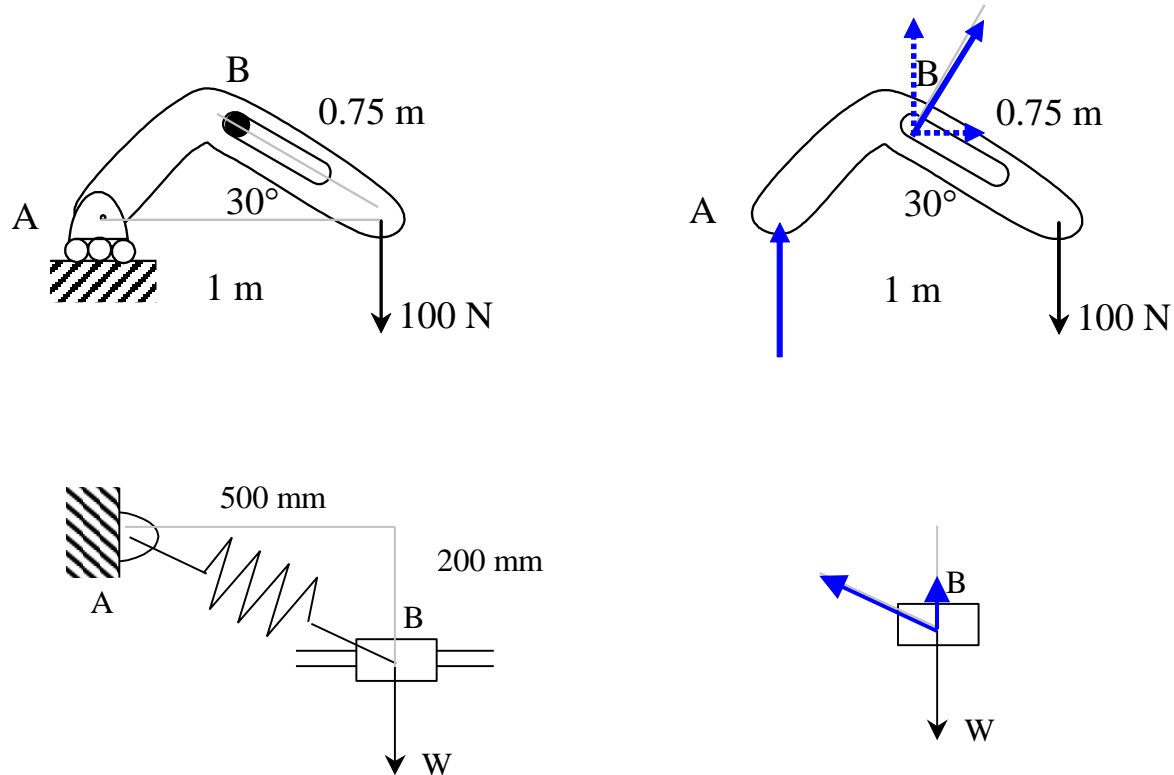


Constraints

- *overconstrained*
 - *won't move*
 - *can't be solved with statics*
 - *statically indeterminate to n^{th} degree*

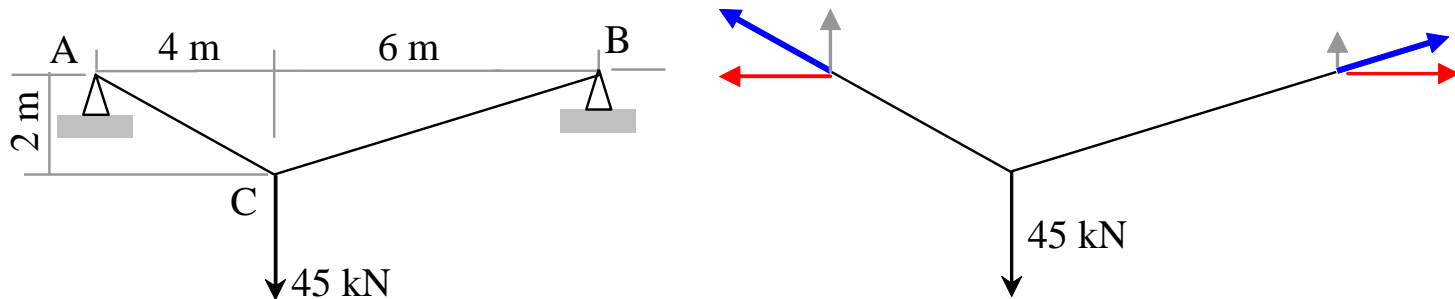


Partial Constraints



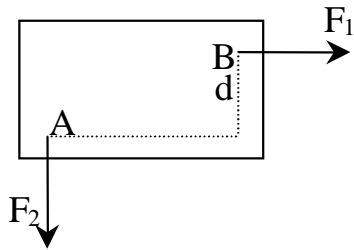
Cable Reactions

- *equilibrium:*
 - *more reactions (4) than equations*
 - *but, we have slope relationships*
 - *x component the same everywhere*

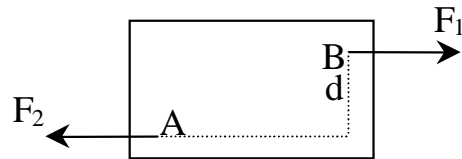


Two Force Rigid Bodies

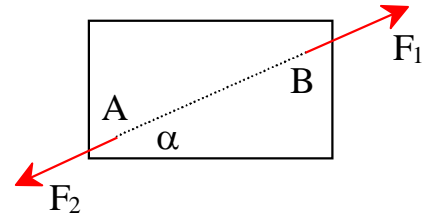
- *equilibrium:*
 - forces in line, equal and opposite



A(no)



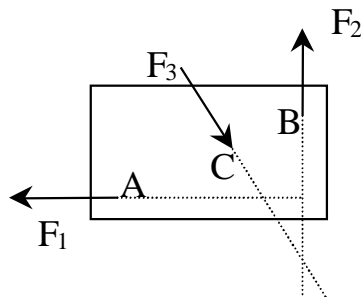
B(no)



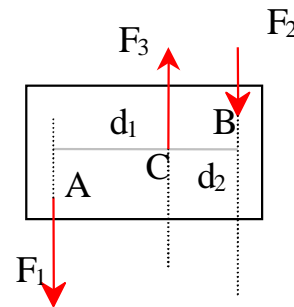
C

Three Force Rigid Bodies

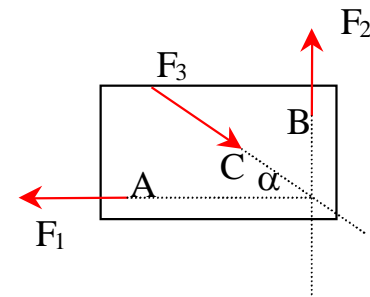
- *equilibrium:*
 - *concurrent or parallel forces*
beams!



A (no)

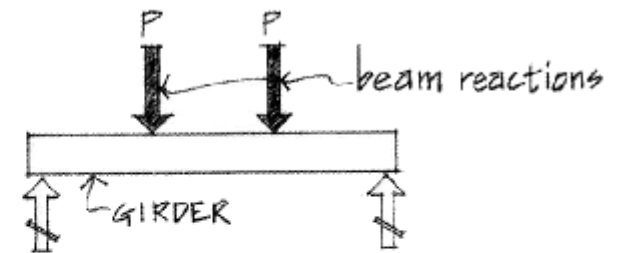
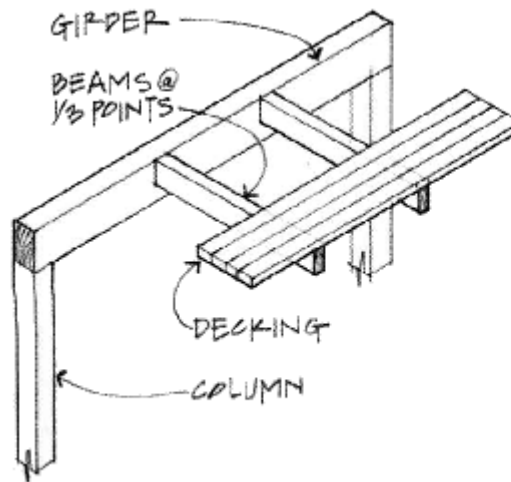
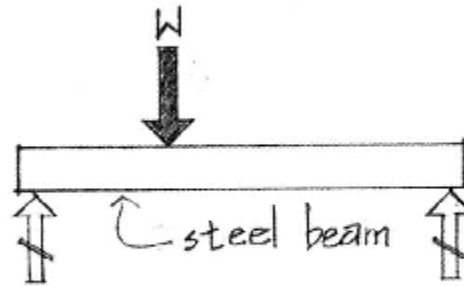
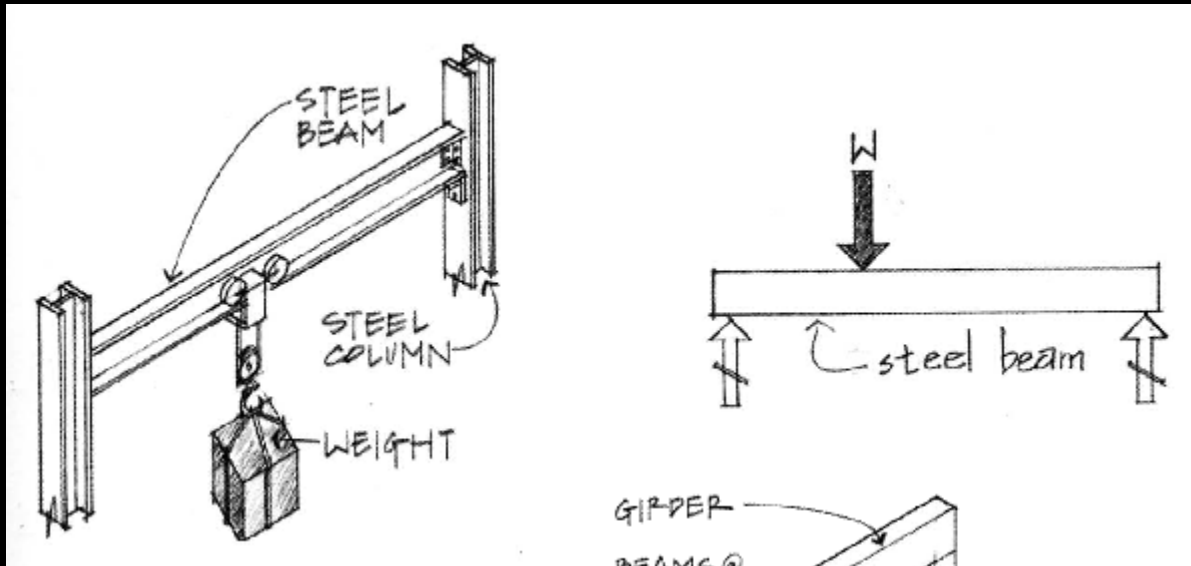


B

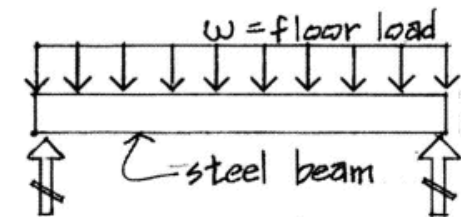
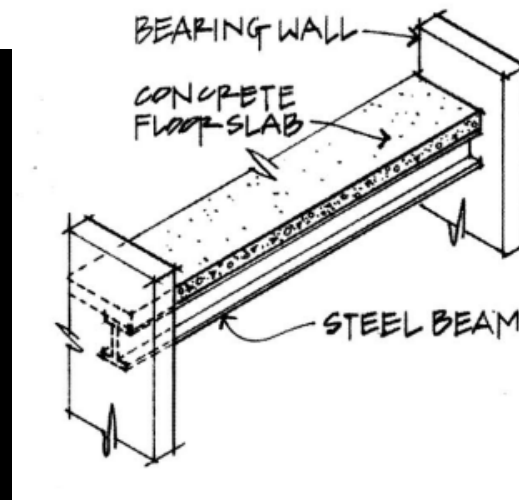
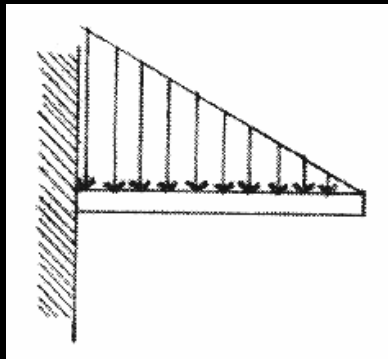
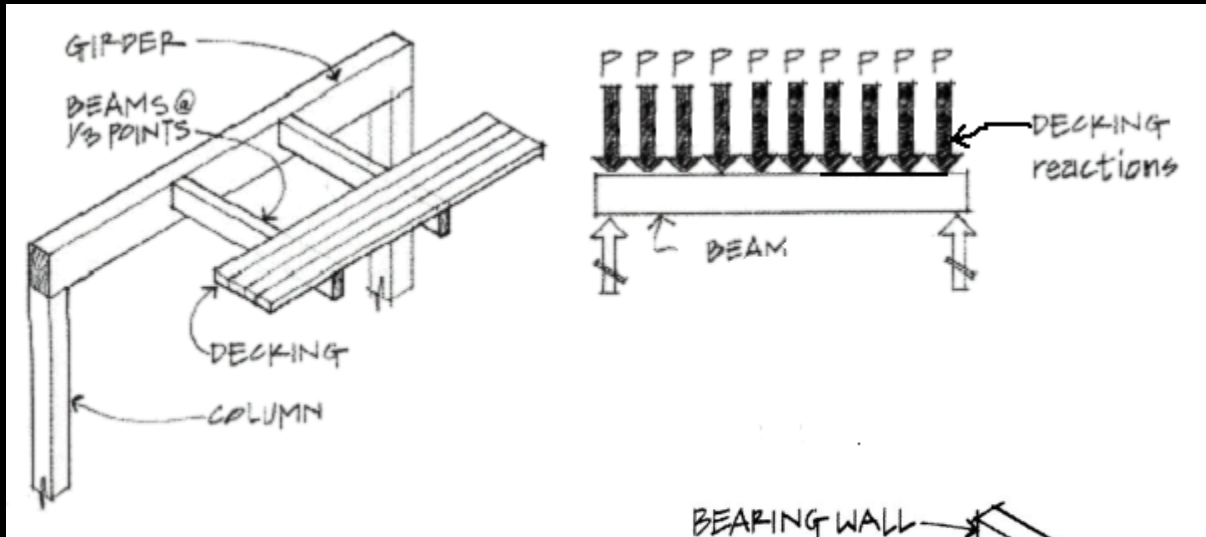


C

Concentrated Loads

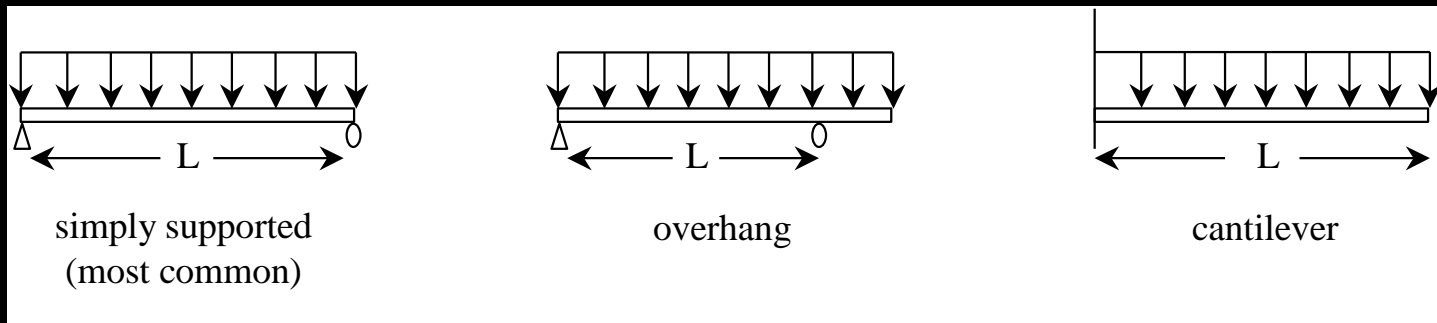


Distributed Loads

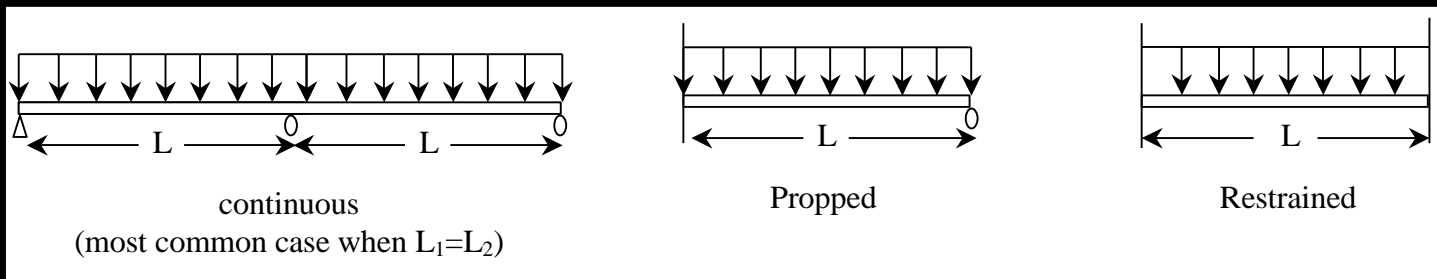


Beam Supports

- *statically determinate*



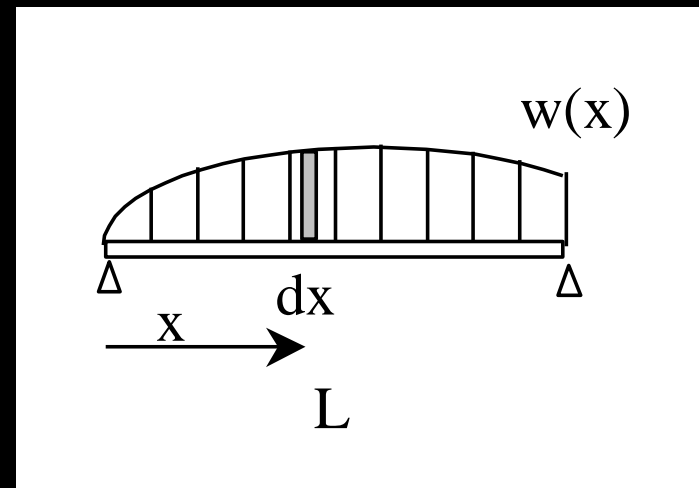
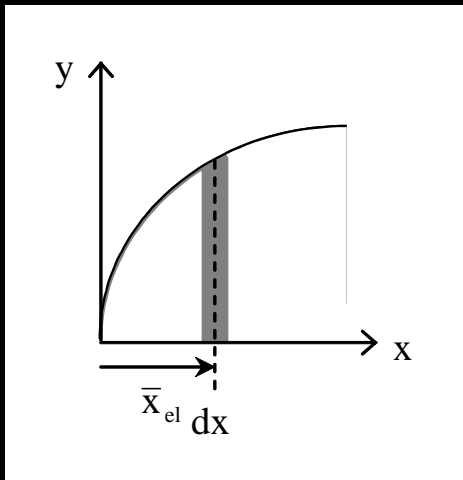
- *statically indeterminate*



Equivalent Force Systems

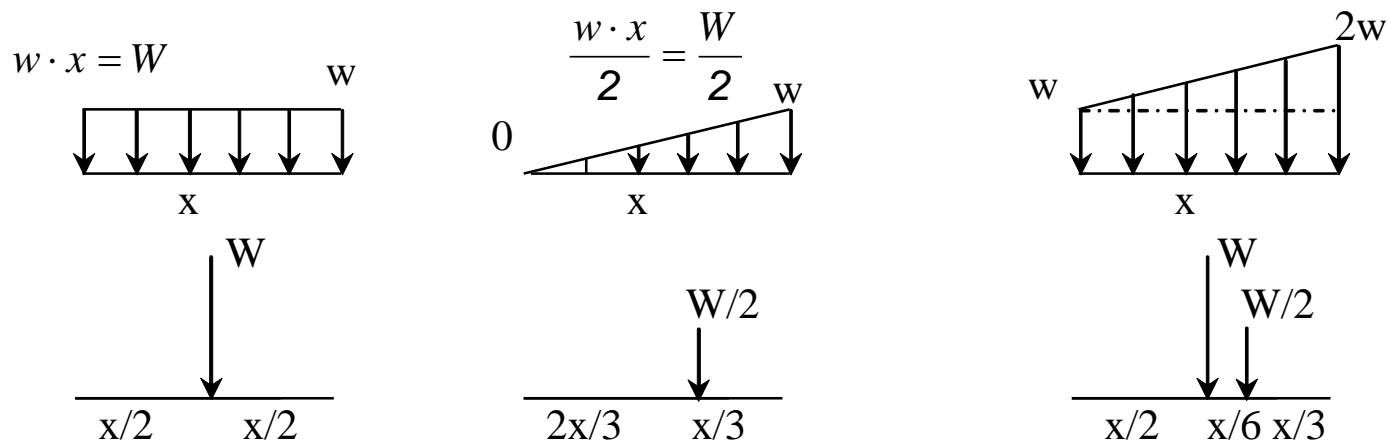
- *replace forces by resultant*
- *place resultant where $M = 0$*
- *using calculus and area centroids*

$$W = \int_0^L w dx = \int dA_{\text{loading}} = A_{\text{loading}}$$



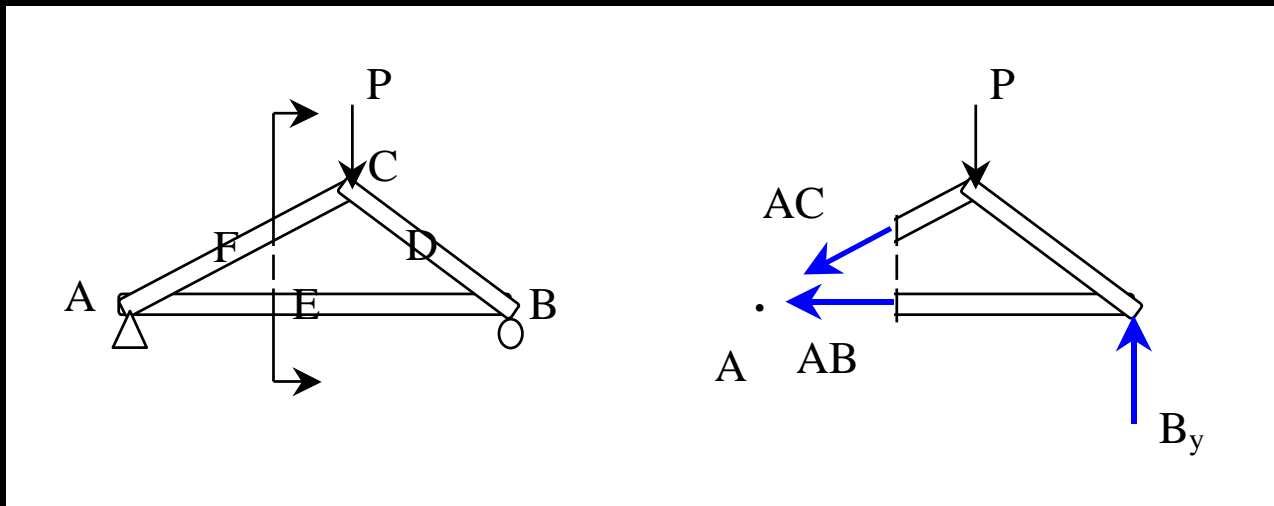
Load Areas

- *area is width x “height” of load*
- *w is load per unit length*
- *W is total load*



Method of Sections

- *relies on internal forces being in equilibrium on a section*
- *cut to expose 3 or less members*
- *coplanar forces $\rightarrow \sum M = 0$ too*



Method of Sections

- *joints on or off the section are good to sum moments*
- *quick for few members*
- *not always obvious where to cut or sum*

