Architectural Structures: Form, Behavior, and Design

ARCH 331

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Lecture five

Mechanics of Materials

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Mechanics of Materials

• MECHANICS

• MATERIALS
Mechanics of Materials

- external loads and their effect on deformable bodies
- use it to answer question if structure meets requirements of
  - stability and equilibrium
  - strength and stiffness
- other principle building requirements
  - economy, functionality and aesthetics
Knowledge Required

- material properties
- member cross sections
- ability of a material to resist breaking
- structural elements that resist excessive
  - deflection
  - deformation

Figure 2.34 An example of torsion on a cantilever beam.
Problem Solving

1. STATICS:
   equilibrium of external forces, internal forces, stresses

2. GEOMETRY:
   cross section properties, deformations and conditions of geometric fit, strains

3. MATERIAL PROPERTIES:
   stress-strain relationship for each material obtained from testing
Stress

- **stress** is a term for the *intensity* of a force, like a pressure
- internal or applied
- force per unit area

\[
stress = f = \frac{P}{A}\]
Design

- materials have a critical stress value where they could break or yield
  - ultimate stress
  - yield stress
  - compressive stress
  - fatigue strength
  - (creep & temperature)
Design (cont)

- we’d like
  \[ f_{\text{actual}} \ll F_{\text{allowable}} \]
- stress distribution may vary: average
- uniform distribution exists IF the member is loaded axially (concentric)
Scale Effect

- **model scale**
  - material weights by volume, small section areas

- **structural scale**
  - much more material weight, bigger section areas

- **scale for strength is not proportional:**
  \[
  \frac{\gamma L^3}{L^2} = \gamma L
  \]
Normal Stress (direct)

- **normal** stress is normal to the cross section
  - stressed area is perpendicular to the load

\[
f_{\text{t or c}} (\sigma) = \frac{P}{A}
\]

Figure 5.7 Two columns with the same load, different stress.
Shear Stress

- stress parallel to a surface

\[ f_v = \frac{P}{A} = \frac{P}{td} \]
Bearing Stress

- stress on a surface by contact in compression

\[
f_p (\sigma) = \frac{P}{A} = \frac{P}{td}
\]

Figure 5.3  Centric loads.
Bending Stress

- normal stress caused by bending

\[ f_b = \frac{Mc}{I} = \frac{M}{S} \]
Torsional Stress

- shear stress caused by twisting

\[ f_v (\tau) = \frac{T\rho}{J} \]
Structures and Shear

- what structural elements see shear?
  - beams
  - bolts
  - splices
  - slabs
  - footings
  - walls
    - wind
    - seismic loads
Bolts

- connected members in tension cause shear stress

- connected members in compression cause bearing stress
**Single Shear**

- seen when 2 members are connected

\[ f_v = \frac{P}{A} = \frac{P}{\pi \frac{d^2}{4}} \]

- (a) Two steel plates bolted using one bolt.
- (b) Elevation showing the bolt in shear.
- (c) (d) A bolted connection—single shear.
Double Shear

- seen when 3 members are connected
- two areas

\[ f_v = \frac{P}{2A} = \frac{P}{A} \frac{1}{2} = \frac{P}{\pi d^2 / 4} \]

Free-body diagram of middle section of the bolt in shear.

Figure 5.12  A bolted connection in double shear.
Bolt Bearing Stress

- compression & contact
- projected area

\[ f_p = \frac{P}{A_{\text{projected}}} = \frac{P}{td} \]
Strain

- materials deform
- axially loaded materials change length
- bending materials deflect

**STRAIN:**
- change in length over length + UNITLESS

\[
\text{strain } \varepsilon = \frac{\Delta L}{L}
\]
Shearing Strain

- deformations with shear
- parallelogram
- change in angles
- stress: $\tau$
- strain: $\gamma$
  - unitless (radians)

\[
\gamma = \frac{\delta_s}{L} = \tan \phi \approx \phi
\]
Shearing Strain

• deformations with torsion
• twist
• change in angle of line

• stress: \( \tau \)
  \[ \gamma = \frac{\rho \phi}{L} \]
• strain: \( \gamma \)
  – unitless (radians)
Load and Deformation

- for stress, need $P$ & $A$
- for strain, need $\delta$ & $L$
  - how?
  - TEST with load and measure
  - plot $P/A$ vs. $\varepsilon$
Material Behavior

- every material has its own response
  - 10,000 psi
  - \( L = 10 \text{ in} \)
  - Douglas Fir vs. steel?

Figure 5.20  Stress-strain diagram for various materials.
Behavior Types

- **ductile - “necking”**
- **true stress**

\[ f = \frac{P}{A} \]

- **engineering stress**
  - (simplified)

\[ f = \frac{P}{A_0} \]
Behavior Types

- **brittle**

- **semi-brittle**
Stress to Strain

- important to us in $f-\varepsilon$ diagrams:
  - straight section
  - LINEAR-ELASTIC
  - recovers shape (no permanent deformation)

Figure 5.20 Stress-strain diagram for various materials.
Hooke’s Law

- straight line has constant slope
- Hooke’s Law

\[ f = E \cdot \varepsilon \]

- \( E \)
  - Modulus of elasticity
  - Young’s modulus
  - units just like stress
Stiffness

- ability to resist strain

- steels
  - same $E$
  - different yield points
  - different ultimate strength

Figure 5.20 Stress-strain diagram for various materials.
Isotropy & Anisotropy

• **ISOTROPIC**
  – materials with $E$ same at any direction of loading
  – ex. steel

• **ANISOTROPIC**
  – materials with different $E$ at any direction of loading
  – ex. wood is orthotropic
Elastic, Plastic, Fatigue

- elastic springs back
- plastic has permanent deformation
- fatigue caused by reversed loading cycles
Plastic Behavior

- ductile

Figure 5.22  Stress-strain diagram for mild steel (A36) with key points highlighted.

at yield stress
Lateral Strain

- or “what happens to the cross section with axial stress”

\[ \varepsilon_x = \frac{f_x}{E} \]

\[ f_y = f_z = 0 \]

- strain in lateral direction
  - negative
  - equal for isometric materials

\[ \varepsilon_y = \varepsilon_z \]
Poisson’s Ratio

- constant relationship between longitudinal strain and lateral strain

\[
\mu = -\frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}
\]

\[
\varepsilon_y = \varepsilon_z = -\frac{\mu f_x}{E}
\]

- sign! \(0 < \mu < 0.5\)
Calculating Strain

- from Hooke’s law
  \[ f = E \cdot \varepsilon \]

- substitute
  \[ \frac{P}{A} = E \cdot \frac{\delta}{L} \]

- get \( \delta = \frac{P L}{AE} \)
Orthotropic Materials

• non-isometric
• directional values of $E$ and $\mu$
• ex:
  – plywood
  – laminates
  – polymer composites
Stress Concentrations

• why we use $f_{ave}$

• increase in stress at changes in geometry
  – sharp notches
  – holes
  – corners

Figure 5.35 Stress trajectories around a hole.
Maximum Stresses

- if we need to know where $f_{\max}^\theta$ and $f_v^\theta$ happen:

\[
\theta = 0^\circ \rightarrow \cos \theta = 1 \quad f_{\max}^\theta = \frac{P}{A_o}
\]

\[
\theta = 45^\circ \rightarrow \cos \theta = \sin \theta = \sqrt{0.5} \quad f_v^\theta = \frac{f_{\max}^\theta}{2A_o} = \frac{f_{\max}^\theta}{2}
\]
Maximum Stresses

**FIG. 2-37** Shear failure along a 45° plane of a wood block loaded in compression

**FIG. 2-38** Slip bands (or Lüders’ bands) in a polished steel specimen loaded in tension
Deformation Relationships

- **physical movement**
  - axially (same or zero)
  - rotations from axial changes

\[ \delta \text{ relates } \delta \text{ to } P \]

\[ \delta = \frac{PL}{AE} \]
Deformations from Temperature

- atomic chemistry reacts to changes in energy
- solid materials
  - can contract with decrease in temperature
  - can expand with increase in temperature
- linear change can be measured per degree
Thermal Deformation

• $\alpha$ - the rate of strain per degree

• UNITS: $^\circ$F / $^\circ$C

• length change: $\delta_T = \alpha(\Delta T)L$

• thermal strain: $\varepsilon_T = \alpha(\Delta T)$

– no stress when movement allowed
## Coefficients of Thermal Expansion

<table>
<thead>
<tr>
<th>Material</th>
<th>Coefficients ($\alpha$) [in./in./°F]</th>
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</thead>
<tbody>
<tr>
<td>Wood</td>
<td>$3.0 \times 10^{-6}$</td>
</tr>
<tr>
<td>Glass</td>
<td>$4.4 \times 10^{-6}$</td>
</tr>
<tr>
<td>Concrete</td>
<td>$6.0 \times 10^{-6}$</td>
</tr>
<tr>
<td>Cast Iron</td>
<td>$6.1 \times 10^{-6}$</td>
</tr>
<tr>
<td>Steel</td>
<td>$6.5 \times 10^{-6}$</td>
</tr>
<tr>
<td>Wrought Iron</td>
<td>$6.7 \times 10^{-6}$</td>
</tr>
<tr>
<td>Copper</td>
<td>$9.3 \times 10^{-6}$</td>
</tr>
<tr>
<td>Bronze</td>
<td>$10.0 \times 10^{-6}$</td>
</tr>
<tr>
<td>Brass</td>
<td>$10.4 \times 10^{-6}$</td>
</tr>
<tr>
<td>Aluminum</td>
<td>$12.8 \times 10^{-6}$</td>
</tr>
</tbody>
</table>
Stresses and Thermal Strains

- if thermal movement is restrained stresses are induced

1. bar pushes on supports

2. support pushes back

3. reaction causes internal stress

\[ f = \frac{P}{A} = \frac{\delta}{L} E \]
Superposition Method

- can remove a support to make it look determinant
- replace the support with a reaction
- enforce the geometry constraint
Superposition Method

- total length change restrained to zero

constraint: $\delta_p + \delta_T = 0$

$$\delta_p = - \frac{PL}{AE} \quad \delta_T = \alpha(\Delta T)L$$

sub:

$$- \frac{PL}{AE} + \alpha(\Delta T)L = 0$$

$$f = - \frac{P}{A} = -\alpha(\Delta T)E$$
Design of Members

• beyond allowable stress…
• materials aren’t uniform 100% of the time
  – ultimate strength or capacity to failure may be different and some strengths hard to test for

• RISK & UNCERTAINTY

\[ f_u = \frac{P_u}{A} \]
Factor of Safety

- accommodate uncertainty with a safety factor:
  \[
  \text{allowable load} = \frac{\text{ultimate load}}{F.S}
  \]

- with linear relation between load and stress:
  \[
  F.S = \frac{\text{ultimate load}}{\text{allowable load}} = \frac{\text{ultimate stress}}{\text{allowable stress}}
  \]
Load and Resistance Factor Design

- loads on structures are
  - not constant
  - can be more influential on failure
  - happen more or less often
  - UNCERTAINTY

\[ R_u = \gamma_D R_D + \gamma_L R_L \leq \phi R_n \]

\( \phi \) - resistance factor
\( \gamma \) - load factor for (D)ead & (L)ive load