beams – internal forces & diagrams
Beams

- span horizontally
  - floors
  - bridges
  - roofs
- loaded transversely by gravity loads
- may have internal axial force
- will have internal shear force
- will have internal moment (bending)
Beams

• transverse loading

• sees:
  – bending
  – shear
  – deflection
  – torsion
  – bearing

• behavior depends on cross section shape
Beams

• bending
  – bowing of beam with loads
  – one edge surface stretches
  – other edge surface squishes
Beam Stresses

• stress = relative force over an area
  – tensile
  – compressive
  – bending

• tension and compression + ...
Beam Stresses

unreinforced concrete beam fails in tension (cracks on bottom)

steel reinforcing in bottom of beam resists tension
Beam Stresses

- tension and compression
  - causes moments

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Beam Stresses

- prestress or post-tensioning
  - put stresses in tension area to “pre-compress”
Beam Stresses

- shear – horizontal & vertical
Beam Stresses

• shear – horizontal & vertical
Beam Stresses

- shear – horizontal

Diagram: Long sand bag (distributed load)

Instructions:
- Draw vertical marker lines before loading to show horizontal shear movement.
Beam Deflections

- depends on
  - load
  - section
  - material

Figure 5.4  Bending (flexural) loads on a beam.
Beam Deflections

• “moment of inertia”
Beam Styles

• vierendeel

• open web joists

• manufactured
**Internal Forces**

- **trusses**
  - axial only, *(compression & tension)*

- **in general**
  - axial force
  - shear force, \( V \)
  - bending moment, \( M \)
Beam Loading

- concentrated force
- concentrated moment
  – spandrel beams
Beam Loading

- uniformly distributed load (line load)
- non-uniformly distributed load
  - hydrostatic pressure = $\gamma h$
  - wind loads
Beam Supports

• **statically determinate**

  - simply supported (most common)
  - overhang
  - cantilever

• **statically indeterminate**

  - continuous (most common case when $L_1 = L_2$)
  - Propped
  - Restrained
Beam Supports

- in the real world, modeled type

(a) Beam supported by a neoprene pad.

(c) Timber beam–column connection with T-plate.
Internal Forces in Beams

- like method of sections / joints
  - no axial forces
- section must be in equilibrium
- want to know where biggest internal forces and moments are for designing
V & M Diagrams

- tool to locate $V_{\text{max}}$ and $M_{\text{max}}$ (at $V = 0$)
- necessary for designing
- have a different sign convention than external forces, moments, and reactions
Sign Convention

- **shear force, V:**
  - cut section to LEFT
  - if $\sum F_y$ is positive by statics, V acts down and is POSITIVE
  - beam has to resist shearing apart by V
Shear Sign Convention

(+) Shear.

(-) Shear.
Sign Convention

- bending moment, $M$:
  - cut section to LEFT
  - if $\sum M_{\text{cut}}$ is clockwise, $M$ acts ccw and is POSITIVE – flexes into a “smiley” beam has to resist bending apart by $M$
Bending Moment Sign Convention

(+) Moment.

(–) Moment.

Holds water

(+)

(–)

Sheds water
Deflected Shape

- positive bending moment
  - tension in bottom, compression in top
- negative bending moment
  - tension in top, compression in bottom
- zero bending moment
  - inflection point
Constructing V & M Diagrams

- along the beam length, plot V, plot M
Mathematical Method

- cut sections with $x$ as width
- write functions of $V(x)$ and $M(x)$
Method 1: Equilibrium

• cut sections at important places
• plot V & M
Method 1: Equilibrium

- **important places**
  - supports
  - concentrated loads
  - start and end of distributed loads
  - concentrated moments

- **free ends**
  - zero forces
Method 2: Semigraphical

- by knowing
  - area under loading curve = change in \( V \)
  - area under shear curve = change in \( M \)
  - concentrated forces cause “jump” in \( V \)
  - concentrated moments cause “jump” in \( M \)

\[
V_D - V_C = - \int_{x_C}^{x_D} w \, dx \\
M_D - M_C = \int_{x_C}^{x_D} V \, dx
\]
Method 2

- relationships

Figure 7.11  Relationship of load, shear, moment, slope, and deflection diagrams.
Method 2: Semigraphical

- $M_{\text{max}}$ occurs where $V = 0$ (calculus)
Curve Relationships

- integration of functions
- line with 0 slope, integrates to sloped

- ex: load to shear, shear to moment
Curve Relationships

- line with slope, integrates to parabola

- ex: load to shear, shear to moment
Curve Relationships

- parabola, integrates to $3^{rd}$ order curve

- ex: load to shear, shear to moment
Basic Procedure with Sections

1. Find reaction forces & moments
   Plot axes, underneath beam load diagram

V:

2. Starting at left

3. Shear is 0 at free ends

4. Shear has 2 values at point loads

5. Sum vertical forces at each section
Basic Procedure with Sections

M:

6. Starting at left
7. Moment is 0 at free ends
8. Moment has 2 values at moments
9. **Sum moments at each section**
10. Maximum moment is where shear = 0!
    (locate where $V = 0$)
Basic Procedure by Curves

1. Find reaction forces & moments
   Plot axes, underneath beam load diagram

V:

2. Starting at left

3. Shear is 0 at free ends

4. Shear jumps with concentrated load

5. Shear changes with area under load
Basic Procedure by Curves

\( M: \)

6. **Starting at left**

7. **Moment is 0 at free ends**

8. **Moment jumps with moment**

9. **Moment changes with area under \( V \)**

10. **Maximum moment is where shear = 0!**
    (locate where \( V = 0 \))
Shear Through Zero

• slope of $V$ is $w$ (-$w$:1)

\[
x \cdot w = V_A \Rightarrow x = \frac{V_A}{w}
\]
Parabolic Shapes

• cases

up fast, then slow
up slow, then fast
down fast, then slow
down slow, then fast
Deflected Shape & $M(x)$

- $-M(x)$ gives shape indication
- boundary conditions must be met
Boundary Conditions

- at pins, rollers, fixed supports: $y = 0$
- at fixed supports: $\theta = 0$
- at inflection points from symmetry: $\theta = 0$
- $y_{\text{max}}$ at $\frac{dy}{dx} = 0$
Tabulated Beam Formulas

- how to read charts

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD

Total Equiv. Uniform Load \[= \frac{wl}{2}\]

R = V \[= \frac{wl}{2}\]

V_x \[= w\left(\frac{l}{2} - x\right)\]

M max. (at center) \[= \frac{wl^2}{8}\]

M_x \[= \frac{wx}{2}(l - x)\]

Δmax. (at center) \[= \frac{5wl^4}{384EI}\]

Δx \[= \frac{wx}{24EI}\left(l^3 - 2lx^2 + x^3\right)\]
Tools

- software & spreadsheets help
Tools – Multiframe

• in classrooms and open access labs
Tools – Multiframe

- **frame window**
  - define beam members
  - select points, assign supports
  - select members, assign section

- **load window**
  - select point or member, add point or distributed loads
Tools – Multiframe

• to run analysis choose
  – Analyze menu
    • Linear

• plot
  – choose options
  – double click (all)

• results
  – choose options